THE

ASTROPHYSICAL JOURNAL

AN INTERNATIONAL REVIEW OF SPECTROSCOPY
AND ASTRONOMICAL PHYSICS

VOLUME XIII APRIL, 1901 NUMBER 3

ON THE ORBITS OF THE *ALGOL* VARIABLES *RR PUPPIS* AND *V PUPPIS.*

By Alexander W. Roberts.

Sufficient observations of the two recently discovered *Algol* variables, *RR Puppis* (*Ch.* 2781) and *V Puppis* (*Ch.* 2852), have been secured at Lovedale to yield, with some degree of exactness, the chief characteristics of their orbital movement.

Both stars are fine examples of the two outstanding types of *Algol* variation: of the type resulting from the revolution of two stars nearly equal in size and brightness, and of the type produced by the revolution of two stars considerably unequal both in size and in brightness.

There is also this added interest, that *V Puppis*, which belongs to the first type of variation, is a spectroscopic binary, and when observations sufficiently refined to yield certain measurements in the line of sight have been obtained, not only the relative, but the absolute dimensions of the system will be known. Any addition to our knowledge of this star is therefore of interest.

RR Puppis lies far beyond spectroscopic reach, at least with its present limitations.

RR PUPPIS. (*Ch.* 2781.)

R. A. $7^h\ 43^m\ 31^s$ (1900)

Dec. $-41°\ 7'.6.$

Observations of this star were begun as soon after the announcement of its variation as possible. The star has accordingly been under observation for the better part of a year, nearly 200 observations having been secured.

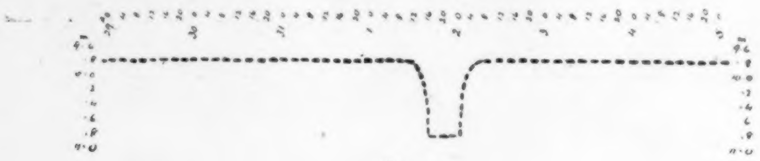


FIG. 1.

The period of variation cannot be far from

$6^d\ 10^h\ 19^m.6.$

With this period all the observations were reduced to the first light curve of 1900.

Figs. 1 and 2 indicate this mean curve. In Fig. 1 is given the whole light curve, and in Fig. 2 the portion of the curve near and at minimum.

The light curve of *RR Puppis* is almost identical in form

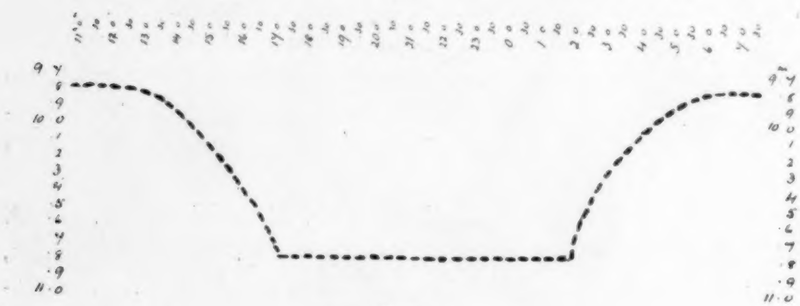


FIG. 2.

with that of *S Velorum*, and naturally the interpretation of the light changes of both stars run on parallel lines.

The data from which the form and relative dimensions of the orbit of *RR Puppis* are deduced may be summarized as follows:

(1) The full period is, as stated, $6^d\ 10^h\ 19^m.6$

(2) The limits of variation are $9^m.1$ and $10^m.8$

(3) The duration of increasing or decreasing phase is $4^h\ 15^m$

(4) The stationary period at minimum is $8^h\ 30^m$.

Statement 3 would indicate that the orbit of *RR Puppis* is practically circular. There is apparently a slight want of correspondence between the increasing and decreasing phases; but this apparent inequality may be due to errors of observation. A series of observations with the new prismatic telescope is being made, with the purpose of settling this difficulty, and also with the hope of obtaining some certain trace of a secondary minimum. As this minimum will not amount to much more than $0^m.05$, the probability of securing unmistakable evidence of its existence is remote.

The presumption of the evidence already obtained is that *RR Puppis* moves in a circular orbit.

Statements 1, 3, and 4 lead to a determination of the relative size of the component stars, assuming a circular orbit.

Thus, let

$$r\ ,\ r_2 = \text{radii of the two stars}\ ;$$

then

$$r_1 + r_2 = \sin \Big\}\, 360^\circ \left(\frac{8^h\ 30^m}{6^d\ 10^h\ 19^m.6} \right)$$
$$= 0.34\ ,$$

the radius of the orbit being equal to unity.

The relation between r_1 and r_2 is given approximately by the ratio

$$\frac{r_1}{r_2} = \frac{4^h\ 15^m}{12^h\ 45^m} = \frac{0.08}{0.26}\ .$$

That is,

Radius of orbit - - - - -	1.00
Radius of *comes* (1) - - - -	0.26
Radius of *comes* (2) - - - -	0.08

The relation between the light intensity of the two stars is readily determined by a simple consideration of the magnitude at maximum and the magnitude at minimum.

At a maximum *RR Puppis* is 2.7 times brighter than it is at a minimum. It is evident that at a maximum we have the combined light of the two stars.

That is, $$L_1 + L_2 = 2.7 \ .$$

At a minimum either *comes* (2), the smaller star, passes *behind comes* (1), and is thus eclipsed by it for 8½ hours, or it transits across it during the same period.

In the latter case we would have

$$L_1 + L_2 = 2.7$$
$$\frac{8}{9} L_1 + L_2 = 1.0 \ ,$$

an inadmissible relation, as in that case L_2 would have a negative value.

In the former case, however, the equation stands

$$L_1 + L_2 = 2.7$$
$$L_1 \qquad = 1.0 \ ,$$

and consequently

$$L_1 = 1.0$$
$$L_2 = 1.7 \ .$$

That is, the smaller star is nearly twice as bright as its larger companion; or, surface for surface, fifteen times more luminous.

As already stated, a similar disparity between size and brightness holds good in the case of the *Algol* variable *S Velorum*.

The relative proximity of two stars so unequal in brightness is indeed remarkable. It is by no means singular, however. And that it is not singular makes the inequality in size and brightness of such systems one of the most interesting problems in stellar physics. Interesting, not only because of its relation to many other allied questions, but because the problem leads to the very threshold of the origin of binary stars.

The solution of the problem will be partially effected when we are able to determine the relative densities of the two component stars. The data at our disposal enables us to say that the density of *RR Puppis* cannot be greater than 0.16, the Sun's density being unity. This result is in complete accordance with what has been ascertained of other close binaries.

Gathering together what we have been able to deduce readily from an examination of the light curve of this star, we find that the system consists of two stars, one three times the diameter of

the other. The smaller star, as in the case of *S Velorum*, is nearly twice as bright as the larger star. The distance between the circumferences of the two stars is about two thirds of the radius of the orbit.

The density of the system cannot be greater than one sixth that of the Sun. It is impossible to say what the relative density of the two components is. It is also not possible from the measures now available to determine the exact form of the orbit or

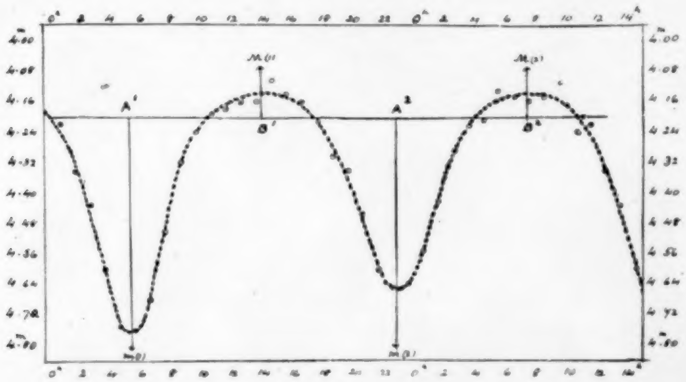


FIG. 3.

its inclination to the plane of sight; but a series of observations are being carried out with the hope of arriving at the value of both these elements.

V PUPPIS. (*Ch.* 2852.)

R. A. $7^h\ 55^m\ 22^s$ (1900)

Dec. —$48^\circ\ 58'.4$

The mean period of this star as determined from a comparison of all available observations is

$1^d\ 10^h\ 54^m\ 26^s.7.$

With this period the observations made at Lovedale during 1899 and the earlier months of 1900 were grouped into thirty-five hourly sets. The mean magnitudes derived from these groups are given in Table I, column 3. If these magnitudes be plotted down and the mean light curve be drawn we have the characteristic form of curve indicated in Fig. 3.

The salient features of this curve, its double and unequal minima, its double and equal maxima, its symmetry, and its close resemblance to the light curve of *U Pegasi* will at once be evident.

The following table gives the observed magnitudes, grouped in thirty-five hourly sets; the magnitude indicated by the mean light curve; the difference between the two, and the number of observations in each set:

TABLE I.

No.	Date (G. M. T.)	Obs. mag.	Mean mag	O-M	No. of obs'ns
1	1900, Jan. 1^d 0^h 27^m	$4^m.22$	$4^m.23$	$-0^m.01$	9
2	1 1 26	4.33	4.33	+0.00	12
3	1 2 30	4.43	4.44	−0.01	13
4	1 3 31	4.60	4.58	+0.02	21
5	1 4 32	4.75	4.76	−0.01	13
6	1 5 25	4.76	4.76	+0.00	19
7	1 6 30	4.68	4.60	+0.08	10
8	1 7 26	4.50	4.46	+0.04	10
9	1 8 30	4.32	4.34	−0.02	12
10	1 9 32	4.24	4.25	−0.01	8
11	1 10 33	4.19	4.19	+0.00	9
12	1 11 28	4.18	4.16	+0.02	13
13	1 12 39	4.16	4.15	+0.01	11
14	1 13 32	4.16	4.14	+0.02	9
15	1 14 34	4.10	4.14	−0.04	11
16	1 15 26	4.14	4.15	−0.01	10
17	1 16 34	4.16	4.16	+0.00	8
18	1 17 28	4.21	4.20	+0.01	8
19	1 18 25	4.30	4.25	+0.05	7
20	1 19 34	4.34	4.33	+0.01	8
21	1 20 28	4.45	4.43	+0.02	10
22	1 21 30	4.60	4.57	+0.03	9
23	1 22 27	4.65	4.65	+0.00	13
24	1 23 35	4.64	4.62	+0.02	13
25	2 0 28	4.51	4.52	−0.01	15
26	2 1 26	4.42	4.39	+0.03	11
27	2 2 20	4.29	4.29	+0.00	7
28	2 3 24	4.22	4.23	−0.01	17
29	2 4 30	4.21	4.18	+0.03	7
30	2 5 32	4.13	4.15	−0.02	10
31	2 6 30	4.15	4.14	+0.01	9
32	2 7 33	4.16	4.14	+0.02	7
33	2 8 23	4.15	4.15	+0.00	8
34	2 9 29	4.11	4.16	−0.05	9
35	2 10 28	4.24	4.18	+0.06	5

The average departure of any set from the mean light curve is only —$0^m.019$, sufficient testimony to the accordance between the observed and mean magnitudes.

In seeking to trace the light variations of this star to the conditions of orbital movement which produce them, we are met at the outset by a grave difficulty. In 1895 Professor Pickering discovered that *V Puppis* is a binary star, the announcement being made in the *Harvard College Observatory Circular* No. 14 as follows:

> An examination of the Draper Memorial photographs taken at the Arequipa station shows that the star *Lacaille* 3105, *Argentine General Catalogue* 10534, is a spectroscopic binary.
>
> The lines in its spectrum were first noticed to be double, and its binary character discovered by the writer (Professor Pickering) in February 1895.
>
> Professor Bailey was notified, and accordingly secured additional photographs and confirmed the binary character of this star.
>
> As in the case of μ' *Scorpii* one component is brighter than the other. A discussion of all the photographs of its spectrum here and at Arequipa gives the mean period $3^d\ 2^h\ 46^m$.
>
> The time of inferior conjunction can be represented by the formula
>
> $$J.\ D.\ 241277.16 + 3^d.115\,E\ .$$
>
> At these times the lines are single; for the next thirty-seven hours the lines are double, the fainter component of each having a greater wave-length than the brighter component and being therefore toward the red end of the spectrum. The lines then again become single and after that for the remainder of the period are again double, the fainter component having shorter wave-lengths and being therefore toward the violet end of the spectrum.

Now the theory which relates light variation to orbital movement requires that the two entirely different lines of investigation, a discussion of the spectroscopic observations, and a consideration of the light changes, should yield precisely the same period, otherwise the relation between variation and revolution is neither intimate nor dependent.

The spectroscopic observations yield a period of $3^d\ 2^h\ 46^m$ and the light observations a period of $1^d\ 10^h\ 54^m.4$. The only connection between these two results is that seven of the former periods are performed in $21^d\ 19^h\ 20^m$ and fifteen of the latter in $21^d\ 19^h\ 36^m$. It is evident therefore that the period obtained from an examination of the variation of the star's light will not satisfy the spectroscopic observations. And if we take the period obtained from the spectroscopic observations and reduce the

magnitude estimates by it to a mean period, the result represents no possible light curve.

I am most reluctant, therefore, to follow out an investigation which apparently has no unequivocal array of facts to justify it. But I am still more reluctant to let a consideration of the variation of this star lie over indefinitely, and, as the only reasonable line along which the investigation can be pursued is that the variation of *V Puppis* is due to orbital motion, and that the period of orbital motion must be, from the nature of things, the period in which a complete cycle of light-changes is performed, this assumption is taken as the starting point of the investigation.

From an examination of the light curve, Fig. 3, we obtain the following phases:

First minimum	(m_1)	1900 January	1^d	5^h	5^m	(G. M. T.)
First maximum	(M_1)		1	13	50	
Second minimum	(m_2)		1	22	40	
Second maximum	(M_2)	1900 January	2	7	15	

From these dates we have

(1)	$M_1 - m_1 =$	8^h 45^m	Diff. from mean	$+ 1^m$
	$m_2 - M_1$	8 50		$+ 6^m$
	$M_2 - m_2$	8 35		$- 9^m$
	$m_1 - M_2$	8 45		$+ 1^m$
	Mean	8 44		

These four approximately equal periods indicate that *V Puppis* passes through each of the four quadrants of its orbit in equal times; that is, the eccentricity must be very small, probably zero.

The excess or defect of the four values given in (1), over or from the mean value 8^h 44^m yield data for a determination both of the eccentricity of the orbit and the position of the line of apsides. The four residuals, however, are well within the margin of probable error and thus it is useless to base a determination of the eccentricity of the system upon values which may have no objective reality.

A graphical indication of the form of the orbit is concurrent with the numerical determination.

Let an abscissa $A^1\ B^1\ A^2\ B^2$ be drawn through the light curve, the direction of the abscissa being along any given magnitude. Join the two minima points, m_1 and m_2, and the two maxima points, M_1 and M_2, with the points on the abscissa midway between the intersections of the abscissa and the light curve. If all the angles at A^1, B^1, A^2, B^2, are right angles, the orbit is circular. If not, then the amount of departure from a right angle is a function both of the eccentricity and apsidal angle. Of course this graphical consideration is only the previous numerical investigation expressed otherwise.

It is evident from an examination of the direction of the ordinates at A^1, B^1, A^2, and B^2, that the orbit of *V Puppis* must be practically circular. Indeed, the nearness of the two component stars, the investigation of which follows later, makes an elliptical orbit an impossibility.

The magnitudes of *V Puppis* at its four principal phases are:

$$
\begin{array}{llr}
M_1 = 4^{\mathrm{m}}.14 & \text{light value} & = 1.00 \\
m_1 = 4.78 & & = 0.56 \\
M_2 = 4.14 & & = 1.00 \\
m_2 = 4.66 & & = 0.62
\end{array}
$$

At M_1 and M_2 we have the combined light of both components. At m_1 we have the whole of the light of one component, which we may call V_2, and a portion, probably, of the light of its companion V_1.

At m_2, V_1 is now unobscured and its neighbor V_2 is either wholly or partially eclipsed.

Let $L_1 =$ light of V_1,
$L_2 =$ light of V_2,
$m =$ portion of V_2 obscured by V_1, when the stars are at m_2,
$k =$ ratio of surface of V_1 to surface of V_2,
$\frac{m}{k} =$ portion of V_1 eclipsed by V_2 when the stars are at m_1.

Then we have the following equations:

$$
\left.
\begin{array}{l}
L_1 + L_2 = 1.00 \\
L_1 + L_2 - mL_2 = 0.62 \\
L_1 + L_2 - \frac{m}{k} L_1 = 0.56 ,
\end{array}
\right. \qquad (2)
$$

which yield the relation:

$$\frac{m}{k} - \frac{0.38}{k} - 0.44 = 0 . \qquad (3)$$

Solutions of this indeterminate equation are conditioned by the fact that $\frac{m}{k}$ and m must both be positive and not greater than unity.

Giving therefore to m such values as will satisfy these conditions we may arrange the dependent values

$$k\,,\ \sqrt{k}\,,\ \frac{m}{k}\,,\ L_1 \text{ and } L_2$$

in the following table:

TABLE II.

m	k	$\sqrt{k}$	$\frac{m}{k}$	L_1	L_2
0.70	0.73	0.85	0.96	0.46	0.54
0.80	0.95	0.98	0.84	0.53	0.47
0.90	1.18	1.09	0.76	0.58	0.42
1.00	1.41	1.19	0.71	0.62	0.38

These values indicate the extreme limits of inequality, both in size and brightness, between the two components V_1 and V_2.

The third column, $\sqrt{k}$, gives the possible values for the ratio of the two diameters. The lowest ratio is 0.85 and the highest, 1.19. That is, the diameter of V_1 is not a fifth less, and cannot be a fifth greater, than the diameter of V_2. We may reasonably infer, therefore, that the two stars which make up *V Puppis* are not markedly unequal in size.

The relative brightness of V_1 and V_2 is also, within certain limits, indeterminate, but these limits are narrow. Under no condition can V_2 be more than one fifth as bright again as V_1, and it cannot be less than two thirds of the brightness of V_1. Whatever possible values we assign to m in equation (3), it appears that surface for surface V_1 is always slightly brighter than V_2.

From the data furnished in Table II we can readily determine the greatest possible inclination, or rather the limits of the inclination. The maximum value of this element is obtained if we consider the two stars to revolve in contact. With this

assumption the greatest possible inclination is 8°; under any circumstance the inclination cannot be less than 5°.

The important bearing of this last conclusion upon spectroscopic observations will be evident when we consider that these yield no data for a determination of the inclination.

An independent solution, therefore, is necessary in order to reduce the spectroscopic measures in the line of sight to movement in the real orbit. With regard to the system *V Puppis* the narrow range of possible inclination enables this reduction to be readily made; indeed, there will be no need for any reduction, as the reducing factor, cos i, can in no case be less than 0.99.

We come now to the relative size of the component stars as compared with the orbit they move in.

In the case of two stars nearly spherical in form, and revolving at some distance apart, the beginning of constant phase will usually be distinctly marked. When the two stars are near enough for their mutual attractions to produce considerable departure from the spherical form, both stars will be more or less pear-shaped, and the exact time when transit begins or ends will be difficult to determine.

Up to this point we have assumed a spherical form for both V_1 and V_2.

If, however, the stars revolve in close contiguity there must be distortion to such an extent as to modify slightly the form of the light curve. But whatever be the amount of distortion, unless actual contact takes place, there will be a stationary period.

On the other hand, if the stars are near enough for their mutual attractions to form a nexus between them, then there will be no stationary period, but the light curve at maximum will be rounded, the sharpness of the curve depending on the oblateness of the stars.

A simple examination of the light curve of *V Puppis* indicates that there is no stationary period at either maximum, and accordingly we must infer that the two component stars revolve around one another *in actual contact.*

In this case there must be considerable distortion in the form of the two stars, especially at the point where the two bodies meet.

It is not possible, although an attempt has been made, to determine the amount of this distortion; the conditions of the problem are too complex, the nature of the action of the forces to be considered too indefinite, and the data at our disposal too meager to enable us to come to a satisfactory conclusion.

We can best appreciate the nature of the difficulty by considering how near we come to a complete explanation of the variation of *V Puppis* by supposing that it is caused by the revolution of two stars, equal in size, but slightly unequal in brightness.

The orbit of the system is circular, slightly inclined to the plane of sight; and the two components stars revolve round one another in contact, yet preserve their spherical form.

Equations (2) and (3) yield the following numerical values:

by assumption, k = unity,

therefore

$$\begin{aligned} m &= 0.82 \\ \frac{m}{k} &= 0.82 \\ L_1 &= 0.54 \\ L_2 &= 0.46 \end{aligned} \tag{4}$$

and,

Radius of orbit = 1.00
Radius of V_1 = 0.50
Radius of V_2 = 0.50
Inclination of orbit = 8°
Period of *V Puppis* = $1^d\ 10^h\ 54^m\ 27^s$
Epoch of prin. min. = 1900 January $1^d\ 5^h\ 5^m$.

With these values we can compute what the variation of *V Puppis* would be.

In the following table (III) is given for comparison the observed magnitudes as set down in Table I, and the theoretical magnitudes computed from the foregoing elements.

TABLE III.

No.	Date	Obs. mags.	Theoretical mags.	O.—T.	No. of obs'ns
1	1900, Jan. 1^d 0^h 27^m	$4^m.22$	$4^m.22$	$+0^m.00$	9
2	1 1 26	4.33	4.32	+0.01	12
3	1 2 30	4.43	4.43	+0.00	13
4	1 3 31	4.60	4.58	+0.02	21
5	1 4 32	4.75	4.75	+0.00	13
6	1 5 25	4.76	4.76	+0.00	19
7	1 6 30	4.68	4.60	+0.08	10
8	1 7 36	4.50	4.46	+0.04	10
9	1 8 30	4.32	4.34	−0.02	12
10	1 9 32	4.24	4.25	−0.01	8
11	1 10 33	4.19	4.19	+0.00	9
12	1 11 28	4.18	4.16	+0.02	13
13	1 12 39	4.16	4.15	+0.01	11
14	1 13 32	4.16	4.14	+0.02	9
15	1 14 34	4.10	4.14	−0.04	11
16	1 15 26	4.14	4.15	−0.01	10
17	1 16 34	4.16	4.16	+0.00	8
18	1 17 28	4.21	4.20	+0.01	8
19	1 18 25	4.30	4.25	+0.05	7
20	1 19 34	4.34	4.33	+0.01	8
21	1 20 28	4.45	4.43	+0.02	10
22	1 21 30	4.60	4.57	+0.03	9
23	1 22 27	4.65	4.66	−0.01	13
24	1 23 35	4.64	4.58	+0.06	13
25	2 0 28	4.51	4.45	+0.06	15
26	2 1 26	4.42	4.34	+0.08	11
27	2 2 20	4.29	4.25	+0.04	7
28	2 3 24	4.22	4.20	+0.02	17
29	2 4 36	4.21	4.16	+0.05	7
30	2 5 32	4.13	4.15	−0.02	10
31	2 6 30	4.15	4.14	+0.01	9
32	2 7 33	4.16	4.14	+0.02	7
33	2 8 23	4.15	4.15	+0.00	8
34	2 9 29	4.11	4.16	−0.05	9
35	2 10 28	4.24	4.18	+0.06	5

To illustrate graphically the agreement between the observed and computed magnitudes Fig. 4 has been drawn representing the actual observed light curve and the theoretical light curve, computed from the orbital elements given in (4).

The agreement between the two curves is very remarkable; indeed there is only one portion of the light curve where the agreement is not complete.

Now it may be urged that this very agreement militates against the correctness of the results obtained, for the theoretical light curve is based upon the assumption that both stars are

spherical, and this assumption is, of course, incorrect. If we consider V_1 and V_2 to be distorted to the amount indicated by the dotted lines in Fig. 5, then the theoretical curve would not be altered at any point as much as $0^m.02$.

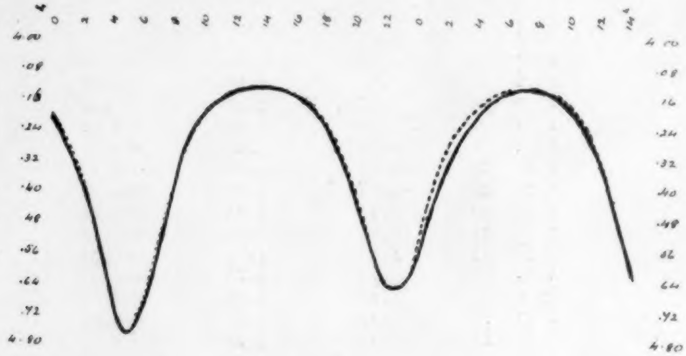


FIG. 4.

It is evident, therefore, that the solution we have obtained does not preclude the possibility of the shape of the two stars being considerably disturbed by their mutual attractions.

Indeed, to introduce this consideration would bring the two curves, the observed and theoretical, into closer agreement. As we have seen, the average departure of a single set of observations from the mean value is $0^m.019$; but the average departure from the theoretical value is $0^m.025$. That is, the theoretical curve can be made to conform still more closely with the observed places.

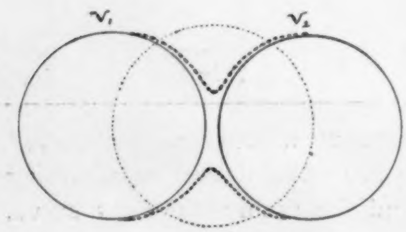


FIG. 5.

In November 1900 a series of observations of *V Puppis* has been begun with the purpose of more thoroughly dealing with the orbit of this star.

Each observation is the mean of four taken with a prism which rotates the field. One observation is taken in each quadrant,

and the mean error of each combined observation will be less than $0^m.03$.

It only remains to consider the density of the system. Applying to the results already obtained any of the formulæ for the determination of the density of close binary stars, we arrive at the result that the density of *V Puppis* cannot be greater than 0.02 of the Sun's density.

The results of the present investigation may be thus summarized:

Period of *V Puppis* - - - -	1^d	10^h	54^m	$26^s.7$	
Principal minimum - - 1900 Jan.	1	5	5		(G. M. T.)
First maximum - - - - -	1	13	50		
Second minimum - - - -	1	22	40		
Second maximum - - - -	2	7	15		
Magnitude at max. - - - -		$4^m.14$			
Magnitude at prin. min. - - -		4.78			
Magnitude at sec. min. - - -		4.66			
Diameter either V_1 or V_2 - - -		1.00			
(Radius of orbit being unity)					
Eccentricity of orbit - - -		0.00			
Inclination of orbit - - - -		8°			
Density of system - - - -		0.02			

LOVEDALE,
December 1900.

ON DOPPLER'S PRINCIPLE.

By W. Michelson.

If n be the number of vibrations per second produced by the source, a, the velocity of the source along a straight line uniting it to the observer, N, the number of vibrations perceived by the observer, and b, the rate at which he moves along the same line, then, according to Doppler's assertion,

$$N = n\frac{v \pm a}{v \pm b}, \qquad (1)$$

where v is the velocity of propagation of the wave-motion in the given medium.

Apart from the assumptions by which it is supported, Doppler's principle has by itself a purely kinematic meaning, and therefore cannot be called in question.

But some of the assumptions on which its application is based are in great measure arbitrary, and can hardly be proved except *a posteriori*, by experimental verification. I shall mention but two of these assumptions: (1) that the period of vibration of the source is not influenced by its motion along the line of sight; (2) that the medium carrying on the waves is at rest as a whole, and that its properties are not changing.

When we have to deal with sound, the source of the waves as well as the medium in all its parts are within our reach. Therefore it is generally easy to test the above mentioned assumptions.

It is quite different when we are observing the displacement of lines in the spectra of celestial bodies. In this case we can neither verify immediately nor prove indirectly either of the assumptions referred to. It is very likely that these displacements are actually due to those motions by which they are usually explained in astrophysics, but, from a strictly logical point of view, it cannot be asserted as yet that no other explanation is possible.

Nor do I mean to remove from Doppler's principle its hypothetical part, which probably belongs to it by the very nature of the question.

All I want is to give it a somewhat different expression in order to comprise under one law also those cases where a change of the frequency is caused not only by the motion of the source or that of the observer, but also by a rapid alteration in the density of the medium crossed by the ray.

It is hardly possible to produce or observe, on a large scale, such rapid changes of density on the Earth, but they not only are conceivable, but very probably take place in the Sun's atmosphere.

Let us return to Doppler's formula (1). As the velocity a and b are generally small compared to the velocity of light, the ratios $\frac{a}{v}$ and $\frac{b}{v}$ are small fractions, whose squares and higher powers can be neglected. Hence the equation (1) can be represented as

$$N = n\frac{1+\frac{a}{v}}{1+\frac{b}{v}} = n\left(1+\frac{a-b}{v}+\ldots\right), \tag{2}$$

reckoning a and b as positive in the direction from the source of vibrations to the observer.

If l be the variable distance of the source from the observer it is evident that

$$b - a = \frac{dl}{dt}$$

is the derivative of the distance to time.

Doppler's formula can then be written as follows:

$$N = n\left(1 - \frac{1}{v}\cdot\frac{dl}{dt}\right). \tag{3}$$

This equation holds also for rays which do not travel in a straight line from the source to the observer, but undergo any number of reflections or refractions on their way. In this case, however, the distance l should be replaced by the optical length of the path of the ray L from the source to the observer.

If the geometrical length of the single parts of the ray's path through different consecutive media be l_1, l_2, l_3, . . . l_n, and the corresponding refractive indices of the media referred to one of them (ether) be μ_1, μ_2, μ_3, ... μ_n, it is obvious that

$$L = l_1\mu_1 + l_2\mu_2 + l_3\mu_3 + \ldots . l_n\mu_n = \sum l\mu ,$$

and Doppler's principle will be expressed by the equation

$$N = n\left[1 - \frac{1}{v}\sum\left(l\frac{d\mu}{dt} + \mu\frac{dl}{dt}\right)\right], \tag{4}$$

where v represents the velocity of light in the medium to which the indices of refraction are referred.

In this equation the additional members of the type

$$\frac{1}{v}\sum\mu\frac{dl}{dt}$$

represent the change of frequency which alone is usually considered in Doppler's principle. This involves also the cases where the length of the ray's path is altered by a rapid displacement of a mirror reflecting it. Mr. W. Wien[1] has made a successful application of a similar change in the period of vibrations produced by reflection from a moving mirror, to the thermodynamics of radiant energy.

These additional members may represent as well the cases where several media of unvarying properties are moving in the path of the rays so as to change rapidly the distance crossed in each of them.

Let us examine two special examples illustrating the case. Suppose that the monochromatic light issuing from the source S passes first through the ether ($\mu_1 = 1$) over the distance l_1, then in a liquid with the index μ_2 over the distance l_2 and then reaches the observer.

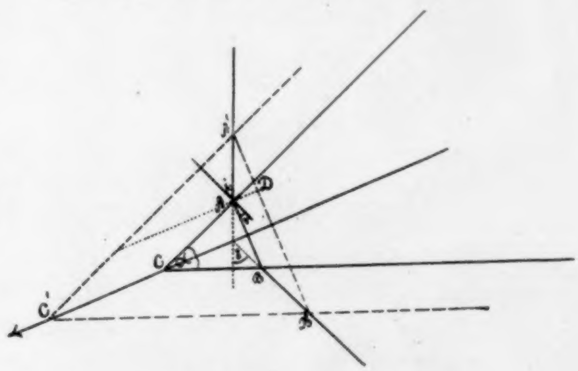
Let l_1 and l_2 change in such a manner that the limiting surface of the ether and the liquid remaining parallel to itself should be displaced rapidly (at the rate c) in the direction of the source.

[1] Willy Wien, *Sitz-ber. d. Berl. Acad.*, **6**, 1893, and *Wiedemann's Annalen*, **52**, p. 156, 1894.

In other words, the thickness of the liquid l_2 increases at the rate c. Then

$$\frac{dl_1}{dt} = -c \; ; \; \frac{dl_2}{dt} = c \; ; \; N = n\left[1 - \frac{c}{v}(\mu_2 - 1)\right]. \qquad (5)$$

If c is commensurable with the velocity of light and μ_2 differs considerably from 1, the displacement of the line can be of the same order as in the case of a rapid motion of the source or the observer. As a second example let us examine the case of a prism moving rapidly across the path of the ray in such a way that the length of the path inside the prism continually increases.



Suppose that the ray passes through the prism at the angle of minimum deviation, which we shall denote by δ. The prism is moving in the direction of CC_1 at the rate c, and its side faces are supposed to be unlimited in their length.

Then let the refracting angle of the prism be	$SCB = \alpha$
the angle of incidence of the ray,	i
the angle of refraction,	r
the refractive index of the prism,	μ
the length of the ray's path inside the prism,	l_2
the length of the ray's path outside the prism,	l_1 ;

then, according to the previous notation,

$$N = n\left[1 - \frac{1}{v}\left(\frac{dl_1}{dt} + \mu\frac{dl_2}{dt}\right)\right]. \qquad (6)$$

If the diagram shows the displacement of the prism in unit of time, then evidently

$$\frac{dl_1}{dt} = -2AA_1 = -2c\frac{\sin\frac{\alpha}{2}}{\cos i} \; ;$$

$$\frac{dl_2}{dt} = +2A'D = 2c\frac{\sin\frac{\alpha}{2}\cos\frac{\delta}{2}}{\cos i}.$$

Putting these formulæ into equation (6) and remembering that

$$r=\frac{\alpha}{2}\ ;\ i=\frac{\alpha+\delta}{2}\ ;\ \mu=\frac{\sin i}{\sin r}\ ;$$

we get

$$N=n\left[1-\frac{2c}{v}\sin\frac{\delta}{2}\right]. \tag{7}$$

The displacement of the spectral line depends only upon the movement of the prism and the angle of deviation of ray δ. If this angle equals 60°, the displacement is equal to that produced by the receding of the source of light at the rate c. If the prism moved in the opposite direction the displacement of the lines should take place in the direction of the violet end of the spectrum.

In spite of the artificiality of these suppositions, some more or less distant analogies must occur in nature. Immediately before a total eclipse of the Moon the Fraunhofer lines in the Moon's spectrum must be displaced toward the red in consequence of the rapid drift of the Earth's atmosphere across the path of the rays. A reverse displacement ought to be observed at the end of totality. A far more considerable displacement must occur immediately before some star is covered by Jupiter, in whose atmosphere the optical path of the rays is considerably lengthened. But one must confess that it will hardly be possible to witness these phenomena by means of observation on account of the strong light of Jupiter himself, as well as on account of the refraction of the rays passing through his atmosphere.

It is not so with the Sun. There phenomena of this kind not only are continually going on, but they have probably been the object of manifold observation, although, to my knowledge, no attempts have as yet been made to explain them in the direction I have indicated. If a velocity of 500 kilometers per second is attributed to the luminous gases of the photosphere and the chromosphere in order to account for the displacement and the distortion of the spectral lines, there is no reason to suppose that higher and non-luminous gases should be less movable. The thinness and sharpness of the Fraunhofer lines show that the gases of the

"reversing layer" are already in a considerably rarefied condition, and are therefore capable of a very swift motion. On the other hand, it can hardly be doubted at present that Schmidt's theory concerning the bearing of the refraction in the Sun's atmosphere is true, at least to a certain extent. Accordingly the rays of the Sun describe probably a very long curvilinear path in the atmosphere before reaching us. This especially concerns the rays issuing from the edges of the solar disk. In this case even comparatively small fluctuations of the solar atmosphere may bring layers of different density into the path of the ray emerging from a certain point of the photosphere or the chromosphere. As "the optical length" of the ray's path may change thereby very rapidly and irregularly, these motions of the non-luminous gases may at least in part account for those extremely rapid and irregular distortions of the spectral lines belonging to Sun-spots and protuberances, which have been observed by Sir N. Lockyer, Professor C. A. Young, and others.

The difficulty of explaining these rapid deviations of the spectral lines in the ordinary way lies less in the assumption of enormous velocities for luminous elements than in the necessity of admitting the existence of inconceivable forces and accelerations which are hardly compatible with the rarefied condition of matter in the Sun's atmosphere. Whereas, according to the explanation I am proposing, a given displacement may be accounted for, in certain cases, also by much smaller velocities; here are acting not only the velocities in the direction of the ray, but also those perpendicular to it. More than this, constant (stationary) velocities may be the cause of most irregular and even opposite distortions of the lines in accordance with the changes of density which take place in the matter carried before the luminous point.

In the case of increasing density in the layers traversed by the ray, the lines are displaced towards the red end of the spectrum, whilst a decrease of density produces a shifting towards the violet end.

Until now it has apparently been considered as an incontrovertible statement that the displacement or distortion of single spectral lines indicates a motion precisely of that matter to which those lines belong. Whereas it becomes evident from the above statements that the motion of hydrogen or helium may have an influence on the displacement of the lines belonging to calcium, iron, etc. It is generally said that if the displacement of the lines depends upon any processes taking place in the path of the rays, and not in the source itself, it would affect not only some of the lines, but all the spectral lines equally. That is true if by "the source of light" one means the whole Sun with the whole of its atmosphere. But since it must be admitted that different elements of this atmosphere may be endowed with velocities of different magnitude and directions—in other words, that they are not mixed,—one must necessarily admit also that the optical paths of the rays issuing from different elements may be different and may be altered almost independently of one another. This may be sufficient to explain in certain cases how the lines belonging to one element may be displaced, while other lines show no disturbances at all.

A strictly scientific solution of the questions, considered from an elementary point of view in this brief account, is hardly possible at present, since it involves the difficult problems of the connection between the ether and ponderable matter.

Moscow,
February 22, 1901.

DIFFRACTION BY LIGHT OF VARIABLE INTENSITY.

By H. M. Reese.

The existence of many spectroscopes in which the absorption of the prisms is considerable makes it of interest to know whether the diffraction pattern due to plane waves passing through an opening of any form is materially altered when the amplitude varies much from one edge of the aperture to the other. The resolving power obtained with such instruments as the Mills spectrograph, in which the light is quite strongly absorbed, would indicate that any such alteration as there may be is not great; but at the same time an analytical investigation seems worth while.

The method employed is merely an extension of that given in the ordinary text-books, such as Preston's *Theory of Light*, for the simpler case of a beam of uniform intensity.

It has been pointed out to the writer by Professor Ames, of Johns Hopkins University, who has kindly criticised this work, that the principal difficulty lies in proving that the waves remain plane after passing through the prisms, or indeed that a wave-front in the ordinary sense of the word exists at all. Consequently, it was thought best to introduce this as a pure assumption. The excellent definition obtained when very dense prisms are used in an ordinary spectroscope seems to show that in such cases at least the ordinary laws of refraction are followed quite closely. In cases such as that of a cyanine prism traversed by light of wave-length nearly that of the absorption-band, the state of affairs may be somewhat different; but it will be shown that the broad, ill-defined image of the slit obtained under these conditions can be explained purely on the basis of diffraction from a plane wave which has a very great range of amplitude, without supposing the wave-surface to have been disturbed at all.

We shall also assume that the absorption by the prisms is proportional to the logarithm of the thickness traversed by the

light. The amplitude of the emergent light at any point will then be proportional to e^{-nl}, where l is the distance of the point from that edge of the beam which passes through the vertex of the prisms. This proportionality will not be changed by reflections from the faces of the prisms, since each reflection merely reduces the amplitude by a constant factor over the whole width of the beam.

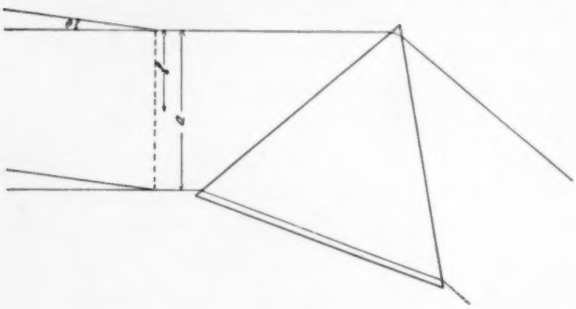


FIG. 1.

Let us suppose the light to be monochromatic, and the aperture to be rectangular, of width a. Fig 2. is the "Cornu spiral" which represents the combined effect, at an angle θ with the wave-normal, of all the elements of the wave-front. Let ds be that element of the spiral which corresponds to an element of wave-front of width dl and of length equal to the dimensions of the aperture in a perpendicular direction. Then $ds = Ae^{-nl}\,dl$. If ϕ is the phase of this element, $\phi = \frac{2\pi}{\lambda}\, l \sin\theta$. If dx and dy are the components of ds, we have

$$dx = \cos\phi\, ds = Ae^{-nl}\cos\left(\frac{2\pi}{\lambda}\, l\sin\theta\right) dl\,,$$

$$dy = \sin\phi\, ds = Ae^{-nl}\sin\left(\frac{2\pi}{\lambda}\, l\sin\theta\right) dl\,.$$

The resulting intensity at the angle θ is represented by $\overline{OM}^2 = x^2 + y^2$.

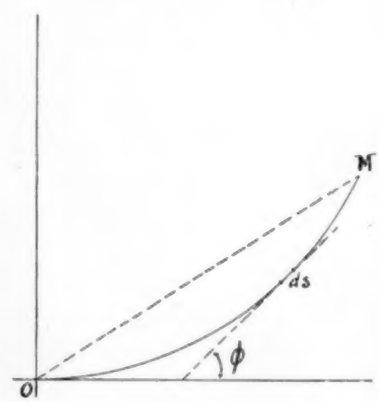


FIG. 2.

To integrate dx and dy let $\frac{2\pi}{\lambda}$ sin $\theta = p$ for convenience. Then, integrating by parts, we get

$$x = A\int_0^a e^{-nl}\cos pl\, dl = \frac{A}{n^2+p^2}\left\{ e^{-na}(p\sin pa - n\cos pa) + n \right\}.$$

$$y = A\int_0^a e^{-nl}\sin pl\,dl = \frac{-A}{n^2+p^2}\left\{ e^{-na}(n\sin pa + p\cos pa) - p \right\}.$$

$$\begin{aligned} I = x^2+y^2 = \frac{A^2}{(n^2+p^2)^2}\Big\{ & e^{-2na}(p^2\sin^2 pa + n^2\cos^2 pa - 2np\cos pa\sin pa) \\ & + e^{-2na}(n^2\sin^2 pa + p^2\cos^2 pa + 2np\cos pa\sin pa) \\ & + 2e^{-na}(np\sin pa - n^2\cos pa) \\ & - 2e^{-na}(np\sin pa + p^2\cos^2 pa) + n^2 + p^2 \Big\}. \end{aligned}$$

$$\begin{aligned} I &= \frac{A^2}{(n^2+p^2)^2}\Big\{ e^{-2na}(n^2+p^2)(\sin^2 pa + \cos^2 pa) - 2e^{-na}(n^2+p^2)\cos pa \\ &\qquad + n^2 + p^2 \Big\} \\ &= \frac{A^2}{n^2+p^2}(e^{-2na} - 2e^{-na}\cos pa + 1) \\ &= \frac{A^2\left\{ e^{-2na} - 2e^{-na}\cos\left(\frac{2\pi a}{\lambda}\sin\theta\right) + 1 \right\}}{n^2 + \frac{4\pi^2}{\lambda^2}\sin^2\theta}. \end{aligned}$$

If we put $n = 0$, this reduces to

$$I = \frac{2A^2\left(1 - \cos\frac{2\pi a}{\lambda}\sin\theta\right)}{\left(\frac{2\pi}{\lambda}\sin\theta\right)^2} = \frac{A^2a^2\sin^2\left(\frac{\pi a}{\lambda}\sin\theta\right)}{\left(\frac{\pi a}{\lambda}\sin\theta\right)^2},$$

which is the same expression arrived at independently. (See Preston, first edition, p. 209.)

For convenience, let us put $\frac{2\pi}{\lambda}\sin\theta = p$, as before, and $e^{-na} = k$. Then we have

$$I = A^2\,\frac{1 + k^2 - 2k\cos ap}{n^2+p^2}.$$

Since $\cos ap$ and p^2 are both even functions of θ, I is symmetrical about the center of the field. It seems at first sight rather strange that a beam of varying intensity should give a symmetrical diffraction pattern, but it can be shown that this will be the case no matter how the amplitude varies, so long we have plane waves to deal with. For the phase difference between the secondary waves coming from any two points in the wave-front is the same in amount for $-\theta$ as for $+\theta$, although opposite in sign, so that the total effect at the angle

$-\theta$ is the same in intensity as that at $+\theta$, but of opposite phase. Another aspect of the same thing is that the Cornu spirals for the two cases are symmetrical to one another with respect to the x-axis.

The expression for I also shows that it is never zero. For in order that it may vanish we must have

$$1+k^2-2k\cos ap=0$$

or

$$\cos ap=\frac{1+k^2}{2k}\,.$$

But $1+k^2$ must be greater than $2k$ unless $k=1$, which means that there is no absorption. Therefore p cannot in general have such a value that I can vanish. It will be shown later, however, that in ordinary cases I does become very small for values of θ which are quite close to those which would make $I=0$ if there were no absorption.

MAXIMA AND MINIMA OF I.

In the first place, if the absorption of the prisms or the width of the beam is so great that $k=e^{-an}$ may be neglected (that is, the intensity of the plane wave at the base of the prism is practically zero), the formula reduces to

$$I=\frac{1}{n^2+p^2}\,,$$

which has no minima, but only a maximum at $p=0$, the intensity becoming less and less toward the edges of the field. This case is perhaps of no practical interest in spectroscopy, but comes close to realization under conditions mentioned above, namely, when a prism showing anomalous dispersion is traversed by light close to its absorption-band. In this case we neglect k not because of the great width of the beam, but because the coefficient of absorption, n, is very great. It is easy to see that under these circumstances the intensity would be quite low at the maximum and would diminish very slowly toward the edges of the field, giving the appearance of a spectral line of extraordinary width and nebulosity

The general condition for a maximum or minimum, $\frac{\delta I}{\delta p}=0$, gives

$$ak(n^2+p^2)\sin ap - p(1+k^2-2k\cos ap)=0\ .$$

This equation is satisfied for $p=0$. It is possible, although somewhat difficult, to determine analytically whether this gives a maximum or a minimum; but it is evidently a maximum from the physical consideration that at this angle all the secondary waves are in the same phase.

To find the secondary maxima and the minima, we might resort to plotting the curves

$$y=\frac{n^2+x^2}{x} \text{ and } y=\frac{1+k^2-2k\cos ax}{ak\sin ax}$$

and determine their intersections, provided we know the numerical values of a and n.

It can be shown analytically, however, that minima exist at $p=\frac{2t\pi}{a}+e$ and maxima at $p=(2t+1)\frac{\pi}{a}+e'$, where t is a positive integer and e and e' are small quantities, the former positive, the latter negative. Of course symmetrical cases occur on the opposite side of the central fringe.

Let $p=s+e$, where $s=\frac{2t\pi}{a}$. Then $ap=2\pi t+ae$. Therefore, if we let ψ represent the function

$$(n^2+p^2)\,ak\sin ap - p(1+k^2-2k\cos ap)$$

we will have

$$\psi=(n^2+s^2+2se+e^2)\,ak\sin ae-(s+e)(1+k^2-2k\cos ae)\ .$$

Since we have supposed e small we may write

$$\sin ae = ae\ ,\ \cos ae = 1-\frac{a^2e^2}{2}$$

and drop terms in e^3. Therefore, approximately

$$\begin{aligned}\psi&=(n^2+s^2+2se)\,a^2ke-(s+e)(1+k^2-2k+a^2ke^2)\\&=a^2kse^2+e\left[a^2k(n^2+s^2)-(1-k)^2\right]-s(1-k)^2\ .\end{aligned}$$

In order that this vanish we must have

$$e=\frac{-P\pm\sqrt{P^2+4a^2ks^2(1-k)^2}}{2a^2ks}\ ,$$

where $P=a^2k(n^2+s^2)-(1-k)^2$.

k is always positive and less than unity, but is not very small unless the absorption is very great indeed. We may say, then, that in most cases P^2 is considerably larger than $4a^2ks^2\ (1-k)^2$, so that we can develop the radical into a power series.

$$\sqrt{P^2+4a^2ks^2(1-k)^2}=P+\frac{4a^2ks^2(1-k)^2}{2P}-\frac{[4a^2ks^2(1-k)^2]^2}{2.4P^3}$$
$$+\frac{1.3[4a^2ks^2(1-k)^2]^3}{2.4.6P^5}-\frac{1.3.5[4a^2ks^2(1-k)^2]^4}{2.4.6.8P^7}$$
$$+\frac{1.3.5.7[4a^2ks^2(1-k)^2]^5}{2.4.6.8.10P^9}-\ldots.$$

Our hypothesis that e is small precludes the use of the negative sign. Therefore

$$e=\frac{1}{2a^2ks}\left\{\frac{4a^2ks^2(1-k)^2}{2P}-\frac{[4a^2ks^2(1-k)^2]^2}{2.4.P^3}+\ldots.\right\}$$
$$=\frac{s(1-k)^2}{P}\left\{1-\frac{a^2ks^2(1-k)^2}{P^2}+\frac{1.3[2a^2ks^2(1-k)^2]^2}{2.3.P^4}-\ldots.\right\}$$
$$=\frac{2\pi t(1-k)^2}{aP}\left\{1-\frac{Q}{\underline{2|}}+\frac{1.3\,Q^2}{\underline{3|}}-\frac{1.3.5\,Q^3}{\underline{4|}}+\frac{1.3.5.7\,Q^4}{\underline{5|}}+\ldots.\right\},$$

where

$$Q=\frac{2a^2ks^2(1-k)^2}{P^2}=\frac{8\pi^2t^2k(1-k)^2}{P^2}.$$

Whenever this series is convergent, and gives a value of e so small that a^3e^3 may be neglected in computing the sine of ae, then $\frac{2\pi t}{a}+e$ gives a critical value of p. It will be shown later that it corresponds to a minimum.

Now, let $p=s'+e'$, where $s'=(2t+1)\frac{\pi}{a}$. Then $ap=(2t+1)\pi+ae'$, $\sin ap=-\sin ae'$, and $\cos ap=-\cos ae'$. Therefore

$$\psi=-(n^2+s'^2+2s'e')a^2ke'-(s'+e')(1+k^2+2k-a^2ke'^2)$$
$$=-a^2ks'e'^2-e'[a^2k(n^2+s'^2)+(1+k)^2]-s'(1+k)^2.$$

In order that this may vanish we must have

$$e=\frac{-P'\pm\sqrt{P'^2-4a^2ks'^2(1+k)^2}}{2a^2ks'},$$

where $P'=a^2k(n^2+s'^2)+(1+k)^2$. For the Mills spectrograph

P'^2 is much greater than $4a^2ks'^2(1+k)^2$, therefore we can develop into a power series as in the former case; and we finally get

$$e' = -\frac{\pi(2t+1)(1+k)^2}{aP'}\left(1 + \frac{Q'}{\underline{2|}} + \frac{1.3\,Q'^2}{\underline{3|}} + \frac{1.3.5\,Q'^3}{\underline{4|}} + \ldots\right),$$

where

$$Q' = \frac{2\pi^2k(2t+1)^2(1+k)^2}{P'^2}.$$

It will be shown that $\frac{(2t+1)\pi}{a} + e'$ corresponds to a maximum.

The expression for $\frac{\delta^2 I}{\delta p^2}$ reduces to

$$\frac{A^2}{(n^2+p^2)^3}\Big[2a^2k\cos ap\,(n^2+p^2)^2 - (n^2+p^2)\Big\{2(1+k^2-2k\cos ap) + 8apk\sin ap\Big\} + 8p^2(1+k^2-2k\cos ap)\Big].$$

Since $\frac{A^2}{(n^2+p^2)^3}$ is always positive we are only concerned with the sign of the other factor. If $p = \frac{2t\pi}{a} + e$, this factor becomes approximately

$$2a^2k(n^2+p^2)^2 - 2(n^2+p^2)\Big\{(1-k)^2 + 4a^2pke\Big\} + 8p^2(1-k)^2,$$

which is always positive, since k is positive and p is large. Therefore we have a minimum in this case.

On the other hand, if $p = (2t+1)\frac{\pi}{a} + e'$, we have approximately

$$-2a^2k(n^2+p^2)^2 - 2(n^2+p^2)\Big\{(1+k)^2 - 4a^2pke\Big\} + 8p^2(1+k)^2,$$

which is negative, corresponding to a maximum.

It is seen that the effect of the absorption is to remove the minima farther from the center of the field, and so widen the central fringe. A short computation shows that in the particular case in hand this widening is very small. A rough experiment gave for n the value 0.12, and the width of the beam, a, is 3.74 cm. This gives $k = .6384$. Putting $s = \frac{2\pi}{a}$, we get

$$P = 25.48$$
$$Q = 0.01015$$
$$\frac{2\pi(1-k)^2}{aP} = 0.00862.$$

This makes $e = 0.00858$. The increase in width of the central fringe is twice this amount, or one half of 1 per cent. of the whole width. A calculation of the intensity at the minimum shows that it is about one half of 1 per cent. of that at the principal maximum.

To determine the resolving-power of any combination of prisms we may plot the intensity-curve of its diffraction pattern

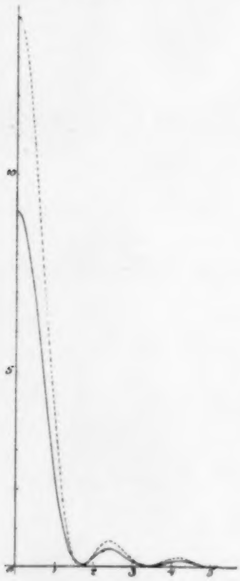


FIG. 3.

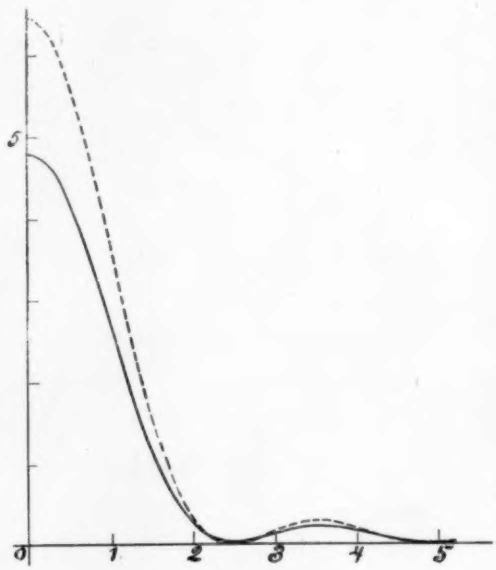


FIG. 4.

and superpose two such curves with the least lateral displacement that will give a combined effect of two maxima distinguishable from one another, as in the ordinary case.

The following considerations will show that the resolving-power of the Mills spectrograph is diminished by less than one half of 1 per cent. by the absorption of the prisms. Let us suppose the diffraction-patterns for λ and $\lambda + \delta\lambda$ to be superposed, $\delta\lambda$ having such a value that the maximum of one falls on the minimum of the other, that is, so that the lateral displacement of the two curves is about one half of 1 per cent. greater

FIG. 5.

than for two resolvable lines in the case of no absorption. We then find that the combined intensity midway between the two principal maxima is $\frac{8097}{10000}$ that at either of the latter, instead of $\frac{8106}{10000}$, as in the corresponding case of no absorption. Therefore the two lines are more than resolved according to the usual definition of resolution.

It was not thought worth while to investigate the secondary maxima any further, but they are well shown in Figs. 3, 4, and 5, representing intensity curves under different conditions. Fig. 3 is for the case already considered where $n=0.12$ and $a=3.74$ cm (about 1½ inches), while Fig. 4 is for $n=0.12$ and $a=2.54$ cm (1 inch), and Fig. 5 is for $n=0.12$ and $a=5.08$ cm (2 inches). In each case the abscissa is p, the ordinate I. The dotted curves show the corresponding intensities when there is no absorption.

LICK OBSERVATORY,
UNIVERSITY OF CALIFORNIA,
February 1901.

A GRAPHICAL STUDY OF REFRACTION AND DISPERSION.

By A. DE GRAMONT.

THE purpose of the following paper is to represent graphically the deviation and dispersion produced by a prism, and to study the properties of the curves thus obtained.

For any one wave-length, *angles of incidence* are used as ordinates, and *deviations* as abscissas. These coördinates have been determined both by experiment and by computation.

EXPERIMENTAL.

Of the several 60° flint prisms which were employed, one which we shall designate as "No. 3 W" had been ground and polished with great care by M. Werlein, so that the interference fringes which it gave by reflection were perfectly straight. This prism illustrates the close agreement between computation and observation.

The measures were made on a circle of 28 centimeters diameter, reading to minutes by means of verniers. The refracting edge of the prism coincided with the vertical axis of the circle. Refractive indices were measured by the minimum-deviation method. The wave-lengths employed were distributed throughout the entire visible spectrum, and were obtained from the following sources: sodium in the flame, hydrogen in Plücker-tube, aluminium, zinc, silver, tin, and lead in the condensed spark.

These results are summarized in the following table, and in the accompanying curve:

TABLE OF REFRACTIVE INDICES.

Wave-lengths	Element	Prism No. 3 W	Prism No. 1
6563	*H*	1.6447	1.6272
5893	*Na*	1.6497	1.6322
5466	*Ag*	1.6543	1.6361
5209	*Ag*	1.6576	1.6395
4861	*H*	1.6630	1.6411
4811	*Zn*	1.6637	1.6461
4680	*Zn*	1.6663	1.6486
4525	*Sn*	1.6697	1.6515
4387	*Pb*	1.6733	1.6550
4341	*H*	1.6745	1.6560
4247	*Pb*	1.6770	1.6586
4058	*Pb*	1.6829	1.6645
3962	*Al*	1.6865	1.6673
3944	*Al*	1.6872	1.6682
Density of prism	..	3.91	3.80
Refracting angle	..	59° 59′	59° 58′

Angles of incidence and deviations.—In order to obtain the deviations of different rays corresponding to a series of different angles of incidence, it is necessary to know the exact reading of the collimator on the divided circle. For this purpose I employed the method of Cornu, which is applicable to both prisms[1] and gratings.[2] An unsilvered mirror is used to throw a beam of light through the slit of the collimator while the circle and the rigidly attached prism are rotated until the image of the slit, reflected

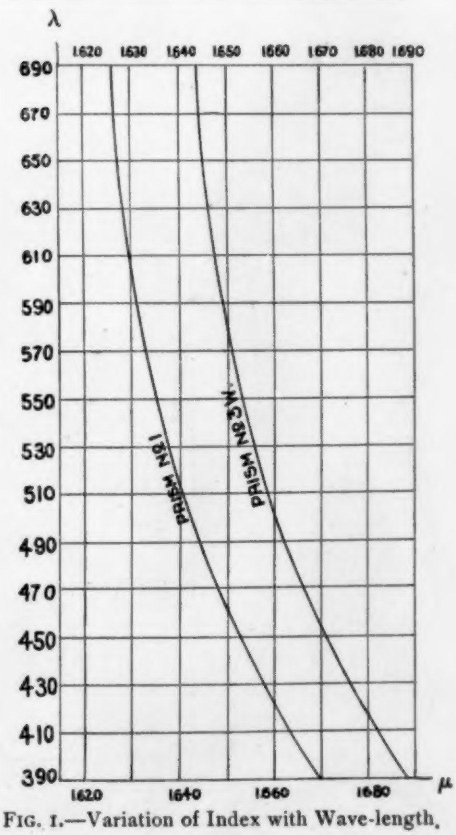


FIG. 1.—Variation of Index with Wave-length.

[1] *Annales de l'École normal supérieure*, 2e série, t. IX.

[2] Etudes sur les bandes telluriques du spectre solaire" (*Annales de Chimie et de Physique*, 6e série, t. VII, p. 48. 1886.

Angles of incidence measured from normal		Deviations						Dispersion		
								Partial		Total
Measured	Calculated	For $H\alpha$ measured	$\lambda = 6563$, $n = 1.6447$ calculated	For D measured	$\lambda = 5893$, $n = 1.6497$ calculated	For Al_2 measured	$\lambda = 3944$, $n = 1.6872$ calculated	$S_{Na} - S_H$	$S_{Al} - S_{Na}$	$S_{Al} - S_{Na}$
Limiting	39° 04′ 41″	....	69° 05′ 41″	....	....	....	....	....	....	....
Limiting	39 28 54	....	....	....	69° 29′ 54″	....	....	....	....	....
40°	....	61° 36′	61 36 30	63° 43′	63 43 44	....	....	2° 07′ 14″	....	....
Limiting	42 33 48	....	....	....	....	....	72° 34′ 48″	....	....	....
45°	....	53 45	53 45 58	54 32	54 32 36	61° 35′	61 35 44	0 46 38	7° 03′ 08″	7° 49′ 46″
49° 26′	....	51 28	51 27 44	52 05	52 04 07	57 02	57 02 05	0 36 23	4 57 58	5 34 21
Minimum ...	55 17 45	50 36 30″	....	....	51 07 19	....	55 08 15	0 30 49	4 00 56	4 31 45
Minimum ...	53 33 00	....	50 36 49	51 07	....	....	55 06 37	0 30 11	3 59 37	4 29 48
55° 34′	....	50 37	50 36 49	51 07	51 06 56	55 06	55 06 22	0 30 07	3 59 26	4 29 33
Minimum ...	57 30 00	....	50 42 39	....	51 11 59	55 02	....	0 29 20	3 50 01	4 19 21
60°	....	51 03	51 02 31	51 30	51 30 41	55 09	55 09 22	0 28 10	3 38 41	4 06 51
70°	....	54 19	54 20 10	54 46	54 45 44	58 01	58 01 39	0 25 34	3 15 55	3 41 29
80°	....	60 24	60 24 00	60 48	60 48 37	63 55	63 55 48	0 24 37	3 07 11	3 31 48
88°	....	67 08	67 08 39	67 34	67 33 03	70 38	70 38 01	0 24 24	3 04 58	3 29 22
Grazing	90	....	69 05 41	....	69 29 54	....	72 34 48	0 24 13	3 04 54	3 29 13

at the face of the prism, coincides with the slit itself. The angle of incidence is limited, on the one hand, by "grazing incidence" and, on the other hand, by what may be called the "limiting angle," beyond which the incident ray no longer penetrates the second face of the prism. Within 2° of "grazing incidence" the spectrum is bright enough to permit of settings on the lines.

As a zero for measuring deviations the direct image of the slit was employed.

My study is limited to the following three lines:

The red line of hydrogen, $H\alpha$, - - - - - - - - - $\lambda = 6563$.
The middle of the D lines, - - - - - - - - - $\lambda = 5893$.
The more refrangible of the aluminium double in the violet, - $\lambda = 3944$.

The agreement between observed and computed values is complete to within the limits of errors of observation, that is, approximately one minute.

COMPUTATION OF DEVIATIONS.[1]

As suggested by M. Cornu, deviations are rapidly computed as follows:

Let A = the refracting angle of the prism,
e and e' = the angles of incidence and emergence, respectively,
r and r' = the interior angles at the first and second faces, respectively,
D = the angle of deviation required,
n = the refractive index for the ray considered.

For example, take prism No. 3 W, in which $A = 59° \; 59'$, $e = 45°$, and for sodium light, $n = 1.6497$.

$$\sin r = \frac{\sin e}{n} \left\{ \begin{array}{r} \log \sin e = \bar{1}.8494850 \\ \text{colog } n = \bar{1}.7825967 \\ \hline \log \sin r = \bar{1}.6320817 \end{array} \right. \qquad (1)$$

$$r' = A - r \left\{ \begin{array}{r} A = 59° \; 59' \\ -r = 25 \;\; 22 \;\; 50'' \\ \hline r' = 34° \; 36' \; 10'' \end{array} \right. \qquad (2)$$

$$\sin e' = n \sin r \left\{ \begin{array}{r} \log \sin r' = \bar{1}.7542594 \\ \log n = 0.2174033 \\ \hline \log \sin e' = \bar{1}.9716627 \end{array} \right. \qquad (3)$$

[1] A résumé of what follows was published in *Comptes Rendus*, February 12, 1900.

$$D = e + e' - A \left\{ \begin{aligned} e' &= 69^\circ\ 31'\ 36'' \\ +e &= 45^\circ \\ \hline A + D &= 114\ 31\ 36 \\ -A &= 59\ 59 \\ \hline D &= 54^\circ\ 32'\ 36'' \end{aligned} \right. \tag{4}$$

Special cases.—For minimum deviation D_m we have $e' = e = e_m$, or

$$e_m = \frac{A + D_m}{2} \ . \tag{4a}$$

The case of grazing incidence, or what is geometrically the same,

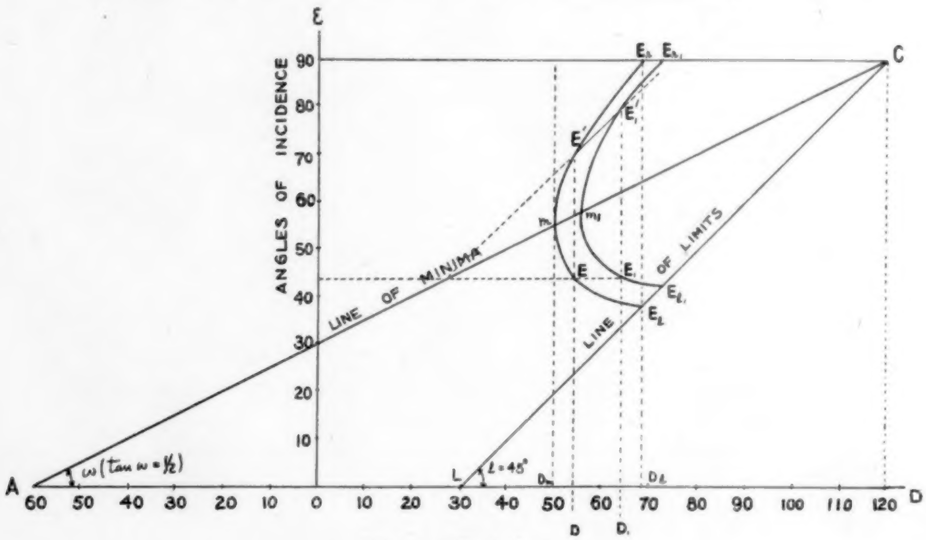


FIG. 2.—Deviations.

the case of grazing emergence, gives us a maximum deviation, viz.:

$$D = 90^\circ + e' - A \ . \tag{4β}$$

The values thus obtained for prism No. 3 W. have been plotted in a curve, Fig. 2, in which deviations are employed as abscissae and angles of incidence as ordinates. The two curves here given are for the red hydrogen line and the violet aluminium line.

Since eq. (4) is symmetrical with respect to e and e', it is evident from the principle of reversibility that to each value of D

there will correspond two values of e, DE and DE' in the figure. The curve thus also gives the values of the angles of emergence.

If now we write eq. (4) as follows,

$$\frac{e+e'}{2} = \frac{1}{2}(A+D)$$

and consider the half sum of e and e' as ordinates, it becomes evident that the locus of the middle points of the chords parallel to the axis of Y lie on a straight line, which we shall call the "line of minima."

This equation shows also that this line makes with the axis of X an angle $\tan^{-1}\frac{1}{2}$, viz., 26° 33′ and that its intercept on the axis of X is numerically equal to the angle of the prism.

Differentiating eq. (4a) we have for any given prism

$$\frac{de_m}{dD_m} = \frac{1}{2},$$

or the minimum deviation varies at a rate which is double that of the angle of incidence.

But from eq. (4) we have, when $e =$ constant,

$$\frac{de'}{dD} = 1.$$

Hence if, on the curves corresponding to two different wave-lengths, we select two points, E and E_1, for which the angle of incidence is the same, we shall find that the points of emergence corresponding to these, E' and E_1' will always lie on a straight line inclined 45° to the axis of X.

For any two given angles of incidence, the ratio of the dispersions will be the ratio of the intercepts between the curves, on the straight lines, $e_1' =$ constant and $e_2' =$ constant, where e_1' and e_2' are angles of emergence corresponding to any given angle of incidence e. This follows at once from the fact that the straight line $E'\ E_1'$, Fig. 2, is inclined 45° to the axis of X.

It is evident also from Fig. 2 that we have a maximum deviation, D_e, determined twice, viz., first by *grazing incidence* and in

the second place, by *limiting incidence*, which gives grazing emergence.

It may also be easily demonstrated, by geometry, that the straight line of minima, the straight line of limits, and the line of grazing incidence, $e = 90°$, all three intersect in one point, C.

Variation of refractive index.— Since in the case of minimum deviation we have

$$\sin e_m = n \sin \frac{A}{2} \ ,$$

it is evident that the maximum permissible value of n is

$$n_e = \frac{1}{\sin \frac{A}{2}} = \operatorname{cosec} \frac{A}{2} \ .$$

In this case maximum and minimum incidence coincide, each having a value of 90°. In other words, our curve of deviations reduces to a point. In the case of a 60° prism,

$$n_e = \operatorname{cosec} 30° = 2 \ .$$

Such a prism can therefore be used only when its refractive index is less than 2.

Let us next consider the curve whose foot is situated at the point L where the line of limits intersects the axis of X. This evidently means grazing incidence and zero angle of emergence; and hence

$$r = A \ ,$$

and

$$n = \frac{1}{\sin A} = \operatorname{cosec} A \ .$$

For a 60° prism, the value of the index giving such a curve is 1.1547. This is a limiting value, for $A = 60°$, of all curves which lie entirely above the axis of X and have positive angles of emergence.

Curves which are partly negative.—We are thus led to consider the case in which $r > A$ and hence $r' < 0$, giving us negative values for the angle of emergence.

The condition that a part of the curve should lie below the axis of X is $n <$ cosec. A. For under this condition the angle of incidence may not only lie anywhere between 0° and 90° but

may even have a negative value and yet give an emergent ray. Total reflection occurs only when these negative values of the angle of incidence exceed a certain limit depending upon the refractive index.

In practice this case is realized only with prisms of very small refracting angle (since the index for liquids does not fall below about 1.33), or with prisms filled with gases optically denser than air.

Variation of refracting angle.—The expression for the maximum permissible index in any prism

$$n = \operatorname{cosec} \frac{A}{2}$$

shows us that n varies inversely as A.

In the case of the extraordinary ray in cinnabar, which yields the highest known index for the red ray, viz., 3.201, the maximum refracting angle is 36° 24′ 30″. Indeed simple inspection of Fig. 2 shows us that, as the refracting angle A of the prism is diminished, the rigidly connected system made up of the straight line of limits and the straight line of minima is displaced farther and farther from the axis of ordinates.

As has been indicated by Cornu, the equation of the curve connecting e and D is given by elimination of e' from the following two equations:

$$e + e' = A + D$$

$$\tan \frac{e + e'}{2} = \frac{\dfrac{\sin^2 \dfrac{A+D}{2}}{\sin^2 \dfrac{A}{2}} - n^2}{n^2 - \dfrac{\cos^2 \dfrac{A+D}{2}}{\cos^2 \dfrac{A}{2}}}.$$

For this latter equation, see Cornu's memoir "De la réfraction à travers un prisme suivant une loi quelconque" (*An. Ec. Norm.* (2), I (1872), p. 240).

Curve of total dispersion.—In Fig. 3, I have plotted, from the data contained in the last column of the table for prism 3 W, a

curve which gives the total dispersion as a function of the angles of incidence, employing as abscissae the differences of deviation between the rays λ 3944 and λ 6563, and using as ordinates the angles of incidence.

This dispersion is a minimum for grazing incidence, increases

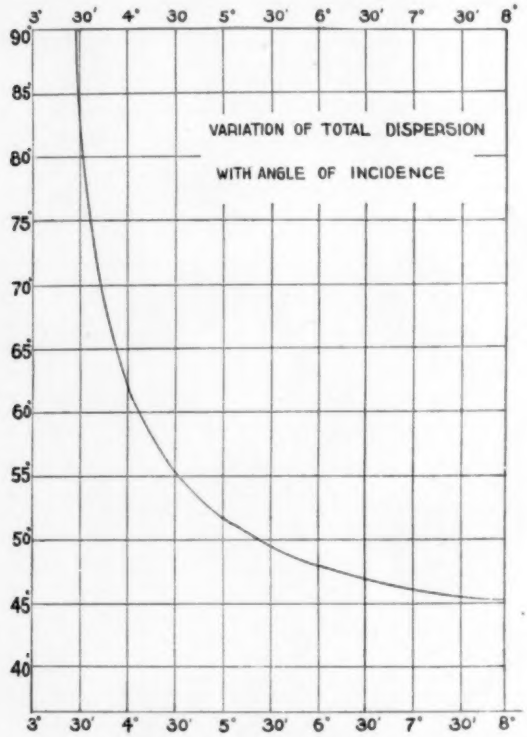


FIG. 3.—Dispersion, $D_{Al_2} - D_{H\alpha}$.

slowly until the minimum of deviation is reached; it then increases rapidly until the limiting angle of incidence is reached. But, as has been indicated above, these limiting angles of incidence depend not only upon the refractive index, but also upon the angle of the prism. Thus we are able to observe the aluminium ray with an index of 1.6872 even up to normal incidence, $e = 0$, provided the angle of the prism is less than 36°. One may thus increase the dispersion in a spectroscope by reducing the angle of incidence.

ON THE SPECTRUM OF *NOVA PERSEI*.[1]

By H. C. Vogel.

The news that a "new star" of considerable brightness had appeared in the constellation Perseus was everywhere received with the highest expectations; for the new stars have ever been among the most puzzling of celestial bodies, and nothing beyond a temporary satisfaction has been furnished by the great number of hypotheses as to the nature of these stars, based upon more or less reliable scientific foundations.

Much obscurity has been cleared up by the application of spectroscopy, and interesting results have ensued in the development of the methods based on Doppler's principle. The perfected apparatus of the most recent date, particularly since the introduction of the spectrograph, has enabled us to recognize in the spectrum of new stars pairs of bright and dark, much broadened lines, which suggested that we are dealing with two bodies instead of one, of which one gives chiefly an emission spectrum and the other an absorption spectrum. The relative motion of the two bodies, deduced from the separation of the centers of the adjacent dark and bright lines, led, however, to velocities so great as to be quite improbable.

The observations made by myself and by others were communicated in detail in my paper "Ueber den neuen Stern im Fuhrmann,"[2] together with the consequences following from them, according to the point of view of that time, and with a discussion of the most important hypotheses.

Our knowledge of the spectra of the different elements has meanwhile been decidedly increased, and the investigations of Humphreys and Mohler, Eder and Valenta, and Wilsing have

[1] From advance sheets of a paper presented to the *Kgl. Akademie der Wissenschaften zu Berlin*, 1901.

[2] *Abhandlungen der Kgl. Akademie zu Berlin.* Translated in *Astronomy and Astro-Physics*, **12**, 896, 1893; **13**, 48, 136, 1894.

taught us that not all displacements of lines are to be regarded as consequences of Doppler's principle. On May 4, 1899, I was able to lay before the Royal Academy a paper by Professor Wilsing of Potsdam entitled "Ueber die Deutung des typischen Spectrums der neuen Sterne,"[1] which gives a very natural explanation of the double spectra of the new stars, based upon his own experiments, and hence of the physical processes effective in the atmosphere of a *Nova*.

We might reasonably expect a further confirmation of the validity of this hypothesis from *Nova Persei*, which we were first able to observe with the spectrograph in Potsdam on February 23, 1901, when it was the brightest star in the northern heavens. Our astonishment was not small to find almost no details visible on the photographs of the spectrum, which with a simple ocular spectroscope was very brilliant.

Measurements by Dr. Hartmann and myself on the spectrograms indicated the presence of the hydrogen lines; on the plates of small dispersion taken by Dr. Hartmann with the 80 cm refractor he could see and measure the nine lines from $H\beta$ to $H\kappa$; while on the plates of high dispersion taken by Dr. Ludendorff with the 32 cm refractor, on which only a small part of the spectrum (from λ 4040 to 4520) was impressed, only the two lines $H\gamma$ and $H\delta$ were to be seen in my examination. The hydrogen lines appeared as broad absorption bands, very weak, diffuse, and only recognizable with difficulty, with an increased tendency to diffuseness of the less refrangible side. Other faint absorption bands of some other element could also be seen, but there was not any suggestion of emission lines or bands. Two quite sharp and narrow absorption lines were, however, very striking on Hartmann's plates, and were identified with the calcium lines at λ 3934 and λ 3969. They indicated a slight displacement toward the red, which according to the preliminary measures would correspond to a motion of the star of about +45 km per second relative to the Earth, or about +18 km per

[1] "On the Interpretation of the Typical Spectrum of the New Stars." The ASTROPHYSICAL JOURNAL, **10**, 113, 1899.

second relative to the Sun. I would remark here that the position of these lines has remained unchanged on the later plates, and the velocity given may probably be regarded as that of the star. On the plates of February 23, which I measured, there were no sharp lines, but there was a somewhat better visible, diffuse absorption band for which I deduced a wave-length of λ 4473. If I identify this band with the helium line λ 4471.6, a velocity of the star of from $+10$ km to $+20$ km, relative to the Sun, would also follow. The case is quite different, however, with the hydrogen bands. Our measures agreed well among themselves and with each other, and yielded a very large displacement toward the direction of shorter wave-lengths, from which could be deduced a velocity of the hydrogen gas of -700 km per second, in round numbers, relative to the Sun.

27 was surely
4481 Mg II
vel − 540 km/sec

Everything that we could observe on February 23 — the strikingly sharp calcium lines, the absence of emission lines, the large displacement of the absorption lines toward the more refrangible side of the spectrum — was contradictory to what was to be expected from the above mentioned theory.

The plates of the spectrum made by Drs. Hartmann, Eberhard, and Ludendorff on February 26 and 27, and on March 2, 3, and 4, with the two instruments mentioned, show a marked change in the spectrum, for the absorption lines had become much more distinct, and were accompanied by intense and very broad emission bands, which could be readily seen as bright lines with the small ocular spectroscope. These emission lines are very broad, gradually becoming diffuse on the less refrangible side; their intensity-maxima are slightly, but their centers greatly, displaced toward red. The absorption lines are, however, shifted still further in the opposite direction than on the spectrograms of February 23. In a word, the spectrum has become that typical of new stars, and shows on a large scale the changes which Wilsing's[2] observations have shown to occur in the spectra of metals and of hydrogen under high pressure.

An attempt to explain the behavior of the calcium lines H

[2] The Astrophysical Journal, **10**, 118, 1899.

and K has led me to interpret the large displacements of the hydrogen lines according to Wilsing's hypothesis.

The rapid increase in the brightness of the new star (which according to Pickering was certainly not of the eleventh magnitude on February 19, and at 10^h on February 23 was of mag. 0.24 according to the Potsdam observations) permits us to assume enormous disturbances in the atmosphere of the star accompanied by heavy increase of pressure—conditions under which we can hardly think of the easily broadened calcium lines as narrow and sharp cut absorption lines; whence we may regard their extreme sharpness as merely an effect of reversal. But if we make this assumption it further follows that a narrow absorption line can exist as a reversal only in a broader emission line, and I assume that layers have developed in the star's atmosphere, as premised in Wilsing's theory, from one of which has resulted broad absorption lines and from the other broad emission bands with reversal effects. If now the superposition of the layers causes absorption and emission to nearly cancel each other, there will remain only the sharp absorption line of the reversal. This is shown in the annexed figure. The upper curve of Fig. 1 *A* represents the intensity curve of the spectrum of emission, with reversal; the lower curve is that of the absorption spectrum. The intensity curve resulting from the superposition of the layers is given in Fig. 1 *B*. The dotted line indicates the position the calcium lines would have if there were no displacement; the narrow absorption line is somewhat displaced toward the red, as observed. In all the diagrams the red end of the spectrum is to be understood as toward the right.

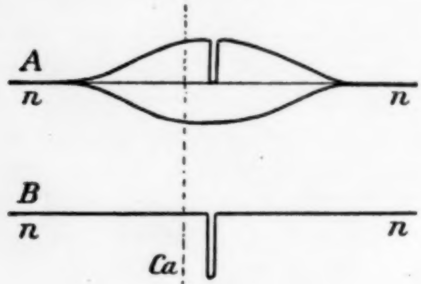


FIG. 1.

Similar considerations remove the difficulty of explaining why the absorption lines of hydrogen only, strongly displaced

toward violet, appeared on February 23. If several layers of different pressure are again superposed, it may easily happen that the emission line, displaced toward red by the higher pressure, is so faint that it does not brighten up the absorption line above the level of continuous spectrum, *n n* in the figures. In Fig. 2 A_1 let the upper curve be that of the emission spectrum, the lower that of absorption; while Fig. 2 B_1 represents the curve resulting from the superposition of the layers, corresponding to the intensity curve observed on February 23. The large displacement of the intensity maximum toward the more refrangible side is therefore only apparent: the center of the absorption band actually needs to be shifted only slightly toward red, as in Fig. 2 A_1.

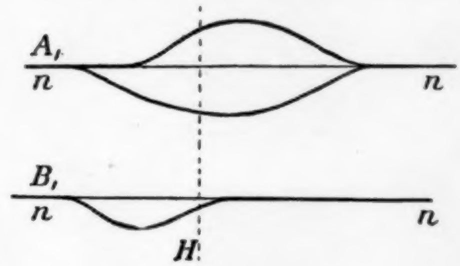


FIG. 2.

As already stated, emission lines are already present on the spectrograms of February 26, and they became increasingly distinct on the later plates. Fig. 3 roughly represents the intensity curve of the $H\gamma$ pair on the plates of March 3 and 4. The peculiar form of the curve is explained as above without difficulty.

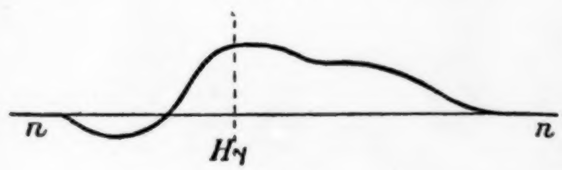


FIG. 3.

I would not fail to mention that these last considerations are only an extension, with inclusion of the phenomena observed in *Nova Persei*, of what Wilsing stated in his paper on the typical spectrum of new stars.

After what has been said I believe I may now express the opinion that the observations of the spectrum of *Nova Persei* have so far only confirmed the view of Wilsing, and that there

is no occasion for regarding the large displacements of the absorption lines of hydrogen as results of motions on Doppler's principle, even if only of motions of hydrogen gas.

The important rôle played by the hydrogen lines in respect to breadth and displacement in case of *Nova Aurigae* seems so far to have been even exceeded in case of *Nova Persei*. There are in this spectrum only a few bands, and no such number of single lines and pairs as in *Nova Aurigae*, and the blue and violet regions are particularly lacking in lines of other elements.

The extremely rapid increase in the star's brightness and now the quick decline in the intensity of the more refrangible parts of the spectrum arouses the suspicion that the hydrogen is chiefly responsible for the bright continuous spectrum in this part, and it is known that this may easily occur under certain conditions of pressure and temperature. The explanation of the rapid decrease in brightness would otherwise be difficult. We must await the further developments of the phenomena, however, before we can reach conclusions with certainty. Perhaps they will lead us to a final discrimination among the many hypotheses as to new stars.

Minor Contributions and Notes

WAVE-LENGTH DETERMINATIONS AND GENERAL RESULTS OBTAINED FROM A DETAILED EXAMINATION OF SPECTRA PHOTOGRAPHED AT THE SOLAR ECLIPSE OF JANUARY 22, 1898.[1]

In this paper the results are given of a detailed study and measurement of a series of spectra photographed at the eclipse of 1898, with a glass prismatic camera of 2¼ inches aperture. Ten exposures were made, all yielding good negatives, in which the great extension in the ultra-violet is a marked feature.

The first two photographs of the series were exposed at 20 seconds and 10 seconds before totality respectively, and are images of the cusp spectrum. They show the Fraunhofer lines with great distinctness, although the latter are much less dark than in the ordinary solar spectrum. The lines were measured and identified for the purpose of facilitating the reduction of the bright line spectra obtained during totality.

Spectrum No. 3 was exposed for four seconds, beginning two seconds before second contact. In this the flash spectrum is fully developed, and extends from λ 3340 to λ 6000. The majority of the bright arcs, including those due to the upper chromosphere, extend over 40° of the limb, implying a depth of 1″.3 for the gases composing this layer. The total depth of the chromosphere deduced from the hydrogen arcs is 8″.2, and from the calcium arcs 11″.6. There are 313 measurable lines in this negative, and the wave-lengths and identifications of these are given in Table I.

Spectrum No. 4, exposed for half a second shortly after second contact, gives the spectrum of the upper chromosphere and prominences. Seven of the latter are shown. The images are about equally dense in calcium radiations, although in hydrogen there is a marked variation of intensity between the different prominences.

[1] Abstract. Read before the Royal Society on January 17, 1901.

A conspicuous feature in the spectrum of two of the prominences is a band of continuous spectrum, beginning at λ 3668 near the end of the hydrogen series, and extending indefinitely in the ultra violet.

Good measures were obtained of the images of a small prominence at the center of the plate, the wave-lengths being given in Table II.

Spectrum No. 5.—This plate had a long exposure near mid-totality. The continuons spectrum of the corona is strongly marked, and the green corona line is well shown at position angles 60° to 78°, and 95° to 105°. A new corona line is faintly impressed at λ 3388 $\pm$, the maxima of intensity being at the same position angles as those of the green line.

Spectrum No. 7 shows the reappearing arcs of the flash spectrum, the exposure ending about four seconds before third contact. The green corona line is shown on both east and west limbs, and there is a faint corona line near H. The wave-length values of the lines measured on this plate are given in Table I.

Spectrum No. 8.—This was exposed almost at the instant of third contact, the reappearing photosphere showing as four narrow bands of continuous spectrum due to Baily's beads. The flash spectrum arcs extend between and across the bands, and can be traced over an arc of 55°, the depth of the layer, in this case exceeding 2".

The focus in this negative is poor, and no measures were made; but as far as can be judged, comparing this plate and No. 3, the spectra of the east and west limbs of the Sun are identical.

Spectra Nos. 9 and 10.—These are cusp spectra, very similar to Nos. 1 and 2.

GENERAL RESULTS AND CONCLUSIONS.

The flash spectrum.—Comparing the wave-length values of the flash spectra given in Table I with Rowland's wave-lengths of the solar lines, it is at once evident that practically all the strong dark solar lines are present in the flash as bright lines; and all the bright lines in the flash, excepting hydrogen and helium, coincide with dark lines having an intensity greater than three on Rowland's scale.

The relative intensities of the lines in the two spectra are, however, widely different, many conspicuous flash lines coinciding with weak solar lines, and some of the strong solar lines being represented by weak lines in the flash spectrum.

This, however, applies only to the spectrum taken as a whole. Selecting the lines of any one element, it is found that the relative intensities in the flash spectrum agree closely with those of the same element in the solar spectrum. This is particularly well shown in the case of the elements iron and titanium.

The want of agreement in the relative intensities of the lines of different elements in the bright line and dark line spectra is probably due to the unequal heights to which the various elements ascend in the chromosphere, a low-lying gas of great density giving strong absorption lines, but weak emission lines, on account of the excessively small angular width of the radiating area.

The more extensively diffused gases of small density, on the other hand, give strong emission lines in the flash spectrum, and weak absorption lines.

The spectrum arcs obtained with a prismatic camera are not true images of the strata producing them, but *diffraction* images more or less enlarged by photographic irradiation. Monochromatic radiations from a layer 2″ in depth will produce arcs or "lines" which are as narrow as can be defined by instruments of ordinary resolving power.

The intensities of these images do not represent the intrinsic intensities of the bright lines of the different elements; the apparent intensity of the radiation from an element depending on the extent of diffusion of that element above the photosphere.

But in the dark line spectrum the intensities depend on the total quantity of each absorbing gas above the photosphere irrespective of the state of diffusion of the different elements.

The flash spectrum as a whole appears from these results to represent the upper, more extensively diffused portion of a stratum of gas, which, by its absorption, gives the Fraunhofer spectrum.

Fifteen elements are recognized with certainty in the flash spectrum (No. 3), and five are doubtfully present. The atomic weights of these elements in no case exceed 91. All the known metals having atomic weights between 20 and 60 seem to be present in the lower chromosphere, but among these there does not seem to be any relation between the atomic weights and the elevations to which the gases ascend in the chromosphere.

The only non-metals found are *H*, *He*, *C*, and possibly *Si*.

Of the 225 lines measured in the ultra-violet region of the spectrum only 29 remain unidentified.

The hydrogen spectrum.—Twenty-eight hydrogen lines are shown in spectrum No. 3. The wave-lengths obtained are compared in Table III with the theoretical values derived from Balmer's formula. With the exception of $H\delta$, which seems to be unaccountably displaced towards the red, the wave-lengths of the ultra-violet lines are found to agree closely with the formula. A slight deviation occurs in the most refrangible lines, the positions of which seem to be distinctly more refrangible than those assigned by theory.

The continuous spectrum given by the prominences in the ultra-violet, beginning at the end of the hydrogen series, seems analogous to a feature noticed by Sir William Huggins in the absorption spectra of first type stars, and is possibly due to hydrogen.

Hydrogen and helium in the lower chromosphere.—From the character of some of the helium lines it is inferred that this element is probably absent from the lowest strata, whilst parhelium appears to be separated from helium, and to exist at a lower level.

Unlike helium, hydrogen gives very intense lines in the flash layer. These lines are well defined and narrow, even in the very lowest strata.

Reasons are given to show that the absence of hydrogen absorption in the ultra-violet, and of helium absorption in the visible spectrum, may be due to insufficient quantity of these elements above the photosphere, not to equality of temperature between the radiating gas and photospheric background.

The corona spectrum.—The wave-length of the green line deduced from measures of No. 3 and No. 7 spectra confirms the value obtained by Sir Norman Lockyer at the same eclipse. The only other lines shown on these photographs are at λ 3388 and near H.

J. EVERSHED.

SIXTY-FOUR NEW VARIABLE STARS.[1]

THE photographs of the Henry Draper Memorial continue to furnish great numbers of new variable stars. A large part of those enumerated below were found from the presence of bright hydrogen

[1] *Harvard College Observatory Circular* No. 54.

lines in their spectra. Many stars whose spectra are of the fourth type also prove to be variable. These variables have been divided into two classes. First, those in which the variation is so great that it is obvious to the most inexperienced observer. Secondly, those in which the variation so far detected is small, about half a magnitude to a magnitude. In each of these cases, two or more experienced observers, who are accustomed to accurate measures of photographic brightness, are satisfied that the change is real. We have here a case like the confirmation visually by a second observer, since so many plates of each variable are examined, generally a dozen or more, that on several the star is bright, and on several, faint. There seems to be no way in which these changes can be rendered more evident, and owing to the redness of many of the stars it is doubtful if visual observations would be more conclusive. Perhaps photometric measures, which appear to be less influenced by color, or photographs taken with a reflector might be employed to advantage. Owing to the accidental errors, additional measures add but little to the certainty of variation, which is best shown by comparing two plates, on one of which the variable is bright, on the other, faint. It seems best therefore to publish the positions of these stars, hoping that by further observations the laws governing their changes may be learned. In both tables, the name of the constellation is given in the first column. For northern stars, the boundary of the constellations is taken from the *Atlas Cœlestis Novus* of Heis, and for southern stars from the *Uranometria Argentina*. The catalogue designation, if any, is given in the second column. The approximate right ascension and declination for 1900 are given in the third and fourth columns. The class of spectrum is given in the fifth column. Following the notation of the *Draper Catalogue*, Mc is used to denote a spectrum of the third type like that of *o Ceti* at minimum. Md denotes a similar spectrum in which, however, the hydrogen lines are bright as in *o Ceti* at maximum. Intermediate spectra are indicated by Mc 5 d. N denotes a spectrum of the fourth type, and Pec. that the spectrum is peculiar. The name of the discoverer is given in the sixth column. A few remarks on individual stars follow Table II. Each is preceded by the right ascension for 1900.

TABLE I.

VARIABLES HAVING LARGE RANGE.

Constellation	Designation	R. A. 1900	Dec. 1900	Class	Discoverer
Chamæleon	*Z. C.* 8^h 2054	8^h $24^m.1$	$-76°$ $2'$	Md	W. P. Fleming
Carina	....	9 18.2	−68 20	Md	W. P. Fleming
Vela	*A. G. C.* 13539	9 51.3	−41 7	N	L. D. Wells
Antlia	*A. G. C.* 14440	10 30.8	−39 3	N	L. D. Wells
Carina	....	10 33.2	−61 48	....	E. C. Pickering
Centaurus	*Z. C.* 11^h 129	11 2.9	−54 35	N	L. D. Wells
Virgo	−18° 3660	13 36.3	−18 38	....	L. D. Wells
Lupus	*Z. C.* 14^h 3225	14 52.6	−53 0	N	W. P. Fleming
Lupus	....	15 8.5	−50 25	....	E. C. Pickering
Circinus	....	15 20.0	−57 22	Md ?	W. P. Fleming
Norma	*Z. C.* 16^h 59	16 2.6	−48 58	Md ?	W. P. Fleming
Norma	*A. G. C.* 21999	16 9.0	−52 21	Md ?	W. P. Fleming
Norma	−51° 10147	16 17.7	−51 42	N	W. P. Fleming
Triang. Austr.	*Gilliss* 12037	16 39.8	−67 36	N	W. P. Fleming
Scorpius	−43° 11672	17 18.1	−43 44	Md	W. P. Fleming
Scorpius	−35° 11923	17 40.8	−35 40	N	W. P. Fleming
Ophiuchus	− 6° 4661	17 44.8	− 6 40	....	W. P. Fleming
Ara	−48° 12145	17 47.3	−48 17	Md	W. P. Fleming
Ara	−49° 11810	17 49.2	−49 46	Mc	W. P. Fleming
Corona Austr.	−39° 12196	17 58.2	−39 20	N	W. P. Fleming
Corona Austr.	....	18 2.6	−45 26	Mc	W. P. Fleming
Corona Austr.	....	18 7.2	−42 53	Md	W. P. Fleming
Telescopium	....	18 19.0	−49 42	Md	W. P. Fleming
Sagittarius	−16° 4904	18 24.6	−16 59	N	W. P. Fleming
Scutum	− 8° 4726	18 44.9	− 8 1	N	L. D. Wells
Scutum	− 8° 4764	18 50.0	− 8 19	N	W. P. Fleming
Sagittarius	....	18 55.9	−12 54	Md	W. P. Fleming
Sagittarius	−22° 4958	18 57.7	−22 51	Mc	W. P. Fleming
Telescopium	....	19 0.5	−49 4	Md	W. P. Fleming
Telescopium	*C. P. D.* −50° 11027	19 10.5	−50 38	Md ?	W. P. Fleming
Lyra	+42° 3338	19 22.2	+42 36	....	W. P. Fleming
Telescopium	....	19 43.1	−50 15	Md	W. P. Fleming
Telescopium	....	20 11.2	−52 56	Md	W. P. Fleming
Telescopium	−51° 12487	20 12.9	−51 1	Mc 5 d	W. P. Fleming
Cygnus	....	21 35.7	+42 45	....	H. R. Colson
Aquarius	−22° 5901	22 17.7	−22 35	Md ?	W. P. Fleming
Piscis Austr.	*A. G. C.* 30605	22 20.5	−29 35	...	W. P. Fleming
Andromeda	+48° 4093	23 28.8	+48 16	Md ?	W. P. Fleming
Pegasus	+25° 5054	23 55.0	+25 21	Md ?	W. P. Fleming

TABLE II.

VARIABLES HAVING SMALL RANGE.

Constellation	Designation	R. A. 1900	Dec. 1900	Class	Discoverer
Hydrus	....	$2^h\ 10^m.4$	$-71°\ 57'$	Mc 5 d	W. P. Fleming
Hydrus	*A. G. C.* 2634	2 26.3	−69 58	Mc 5 d	W. P. Fleming
Cetus	*A. G. C.* 2859	2 37.4	−23 2	Mc 5 d	W. P. Fleming
Horologium	*Z. C.* 2^h 1104	2 41.2	−54 44	Mc	W. P. Fleming
Eridanus	−1° 546	3 46.4	− 1 41	Mc 5 d	W. P. Fleming
Puppis	*A. G. C.* 8954	7 1.7	−35 47	Mc 5 d	W. P. Fleming
Canis Major	−11° 1805	7 3.4	−11 46	N	W. P. Fleming
Lynx	+46° 1271	7 20.9	+46 10	Mc	W. P. Fleming
Hydra	− 8° 2343	8 19.6	− 8 11	Mc 5 d	W. P. Fleming
Hydra	− 9° 2612	8 34.9	− 9 14	Mc 5 d	W. P. Fleming
Virgo	− 8° 3329	12 15.2	− 8 27	Md ?	W. P. Fleming
Centaurus	*A. G. C.* 17944	13 6.3	−56 28	....	L. D. Wells
Virgo	− 2° 3653	13 8.9	− 2 16	Mc 5 d	W. P. Fleming
Chamæleon	*A. G. C.* 18352	13 24.6	−77 3	....	W. P. Fleming
Lupus	*Z. C.* 14^h 970	14 16.9	−47 4	N	W. P. Fleming
Norma	−50° 10442	16 14.6	−50 14	Md	W. F. Fleming
Serpens	−15° 4923	18 13.6	−15 39	N	W. P. Fleming
Corona Austr.	....	18 23.7	−45 2	Md	W. P. Fleming
Telescopium	−48° 12910	19 0.1	−48 44	Mc	W. P. Fleming
Sagittarius	−16° 5360	19 28.6	−16 35	N	L. D. Wells
Sagittarius	*C. P. D.* −41° 9189	19 40.6	−41 26	Mc 5 d	W. P. Fleming
Sagittarius	*A. G. C.* 27520	20 0.8	−27 31	Mc 5 d	W. P. Fleming
Octans	*Gilliss* 15580	22 5.7	−85 10	Mc 5 d	W. P. Fleming
Aquarius	−18° 6299	23 19.2	−17 52	Pec.	W. P. Fleming
Cassiopeiæ	+56° 3111	23 49.4	+56 56	Pec.	L. D. Wells

$7^h\ 20^m.9$ The variation, although small, has been confirmed by two other observers, and is indicated by observations with the meridian photometer.

10 33.2 Found by superposing an original negative on a contact print from another negative taken on a different date.

15 8.5 Found by superposing an original negative on a contact print from another negative taken on a different date.

7 40.8 This star is *C. P. D.* −35°7243. Innes has announced the variability of −35°7270, which follows $51^s.9$, south $0'.1$. *A. J.*, **20**, 59, 95.

18 44.9 "Probably a variable of the *19 Piscium* type" in Espin's Catalogue of Red Stars, *Cunningham Memoirs*, No. V, 75. Discovered also independently by Mrs. Fleming.

19 22.2 Found by inspection of a photograph taken as described in *Circular* No. 29. Thirteen exposures of $29^m\ 40^s$ each were made on July 13, 1899, stopping the clock automatically for 20^s after each exposure. This variable is mentioned in the Fifty-fourth Annual Report. Photometric measures show that its maxima are represented by the formula, *J. D.* $2,414,856^d.500 + 0^d.5668\ E$. Range 0.83 magnitudes.

21 35.7 Discovered visually during observations of *SS Cygni*.

23 49.4 This star is *ρ Cassiopeiæ*. The variation, although small has been confirmed by four other observers. The spectrum closely resembles that of the second type.

Measures have been made of a number of the stars in the above tables and also of those announced without magnitudes in previous circulars. The right ascension and declination for 1900, the number of plates examined, and the brightest and faintest photographic magnitudes, are given in the successive columns of Table III. The last column gives the authority for the variability.

TABLE III.

PHOTOGRAPHIC MAGNITUDES.

R. A.	Dec.	No.	Br.	Ft.	Authority	R. A.	Dec.	No.	Br.	Ft.	Authority
2^h $10^m.4$	−71° 57′	75	9.6	10.5	Table II	16^h $54^m.3$	−54° 55′	50	9.9	11.0	*Circular* 24
2 37.4	−23 2	68	7.7	8.6	Table II	17 34.7	−57 40	43	8.3	9.8	*Circular* 24
6 28.1	− 8 48	54	9.0	10.1	*Circular* 32	17 45.7	−51 40	56	8.9	12.4	*Circular* 24
8 1.7	−38 29	47	0.3	10.2	*Circular* 24	17 47.3	−48 17	134	9.7	<12.3	Table I
8 3.1	−22 38	23	9.4	11.6	*Circular* 24	18 13.6	−15 39	...	8.5	9.1	Table II
8 24.7	− 5 59	48	8.0	9.6	*Circular* 24	18 19.0	−49 42	129	11.3	<12.7	Table I
8 34.9	− 9 14	70	7.7	9.0	Table II	18 23.7	−45 2	107	11.9	12.6	Table II
9 13.5	−65 49	116	10.9	12.1	*Circular* 32	19 10.5	−50 38	113	9.2	10.6	Table I
10 8.3	+60 31	76	7.0	8.3	*Circular* 32	19 22.2	+42 36	191	7.2	8.1	Table I
10 33.2	−61 48	210	10.1	<12.5	Table I	19 37.1	+32 23	58	8.7	10.3	*Circular* 24
11 59.6	− 5 13	77	7.2	8.8	*Circular* 32	20 3.3	−60 14	75	9.0	10.2	*Circular* 24
12 2.1	− 6 12	80	7.1	8.3	*Circular* 24	20 11.2	−52 56	91	10.5	12.9	Table I
13 15.1	−61 3	55	10.5	11.3	*Circular* 24	20 12.9	−51 1	124	8.1	9.7	Table I
14 1.7	+13 59	30	10.0	13.0	*Circular* 24	21 13.6	−45 27	83	7.2	8.9	*Circular* 24
16 17.7	−51 42	102	11.0	<12.3	Table I	23 19.2	−17 52	73	8.3	< 9.4	Table II

EDWARD C. PICKERING.

January 24, 1901.

THE SPECTRUM OF *ζ PUPPIS*.[1]

THE presence of a second series of hydrogen lines, in addition to the ordinary series, in the spectrum of *ζ Puppis*, was announced in *Circulars* Nos. 12, 16, and 18. Accurate wave-lengths could not then be determined for the less refrangible lines. Since then, measures have been made of six photographs of spectra of *ζ Puppis*, and two of spectra of *δ Orionis*. The measurements have been made by Miss F. Cushman, and the conversion into wave-lengths, by Mr. Edward S. King, with the assistance of Miss Cannon. This work will be pub-

[1] *Harvard College Observatory Circular* No. 55.

lished in full in a later volume of the *Annals*. Following the notation proposed by Vogel for the ordinary series of hydrogen lines, the new series may be designated, $H\alpha'$, $H\beta'$, $H\gamma'$, etc. The great difficulty in determining the wave-length of $H\beta'$ which is approximately 5414, is that no known lines of greater wave-length have been found in the spectrum of ζ *Puppis*. Accordingly, extrapolation, which is always uncertain, must be employed. The star, δ *Orionis*, besides the line $H\beta'$, contains also the known line D_3, wave-length 5876, which thus permits the wave-length of $H\beta'$ to be determined by interpolation. These lines also occur in ϵ *Orionis*, and probably in other stars of the so-called *Orion* type. It is greatly to be desired that the spectra of these stars should be photographed with a slit spectroscope attached to a large telescope, as the wave-lengths of these lines could then be determined with far greater accuracy by the help of a comparison spectrum. A comparison of the mean of the wave-lengths given in *Circular* No. 16, with those recently determined, and with two computed values, is given in the following table. The first of these is given in *Circular* No. 16, and is a slight modification of Balmer's formula. It is $3646.1 \frac{n^2}{n^2 - 16}$, in which n is an even number for the ordinary series of hydrogen lines, and an odd number for the additional series. The second formula is $\frac{1}{\lambda} = A + B\frac{1}{m^2} + C\frac{1}{m^4}$. This form was proposed by Kayser, immediately after the announcement of the discovery of this series of lines. If we accept this formula, it would appear to be the true law connecting their wave-lengths, and would render them comparable with those of other elements. The designation of the lines is given in the first column of the following table, the mean of the observed wave-lengths given in *Circular* No. 16 is contained in the second column, and the wave-lengths derived by Mr. King in the third column. The next four columns give the value of n, taken from the sixth column of the table in *Circular* No. 16, and computed wave-lengths taken from the seventh column of the same table, and the residuals found by subtracting the computed values from the observed values given in the second and third columns. A similar comparison with the formula $\frac{1}{\lambda} = 27461 - 121790\frac{1}{m^2} - 352010\frac{1}{m^4}$, is contained in the last four columns of the table.

Des.	Clr. 16	Obs.	n	Comp.	O–C	O–C	m	Comp.	O–C	O–C
$H\alpha'$	......	......	5	10128.1	.....	.....	3	10435.2	.....	.....
$H\beta'$	......	5413.6	7	5413.9	.....	−0.3	4	5413.0	.....	+0.6
$H\gamma'$	......	4542.4	9	4543.6	.....	−1.2	5	4540.1	.....	+2.3
$H\delta'$	4200.6	4200.7	11	4201.7	−1.1	−1.0	6	4200.6	0.0	+0.1
$H\epsilon'$	4026.3	4026.0	13	4027.4	−1.1	−1.4	7	4027.5	−1.2	−1.5
$H\zeta'$	3924.8	3924.0	15	3925.2	−0.4	−1.2	8	3925.8	−1.0	−1.8
$H\eta'$	3858.6	3860.8	17	3859.8	−1.2	+1.0	9	3860.6	−2.0	+0.2
$H\theta'$	3816.0	3815.7	19	3815.2	+0.8	+0.5	10	3815.4	+0.6	+0.3
$H\iota'$	3783.4	......	21	3783.4	0.0	.....	11	3782.1	+1.3	.....
	......	......	∞	3646.1	.....	.....	∞	3641.5	.....	.....

It will be noticed that the two computed values do not differ very greatly for any of the observed lines. The difference is greatest for $H\gamma'$, and here the observed value is nearly midway between the two computed values. On the whole, the observed values agree more nearly with the first formula, than with the second. This is remarkable, if it does not represent the true law, since this formula contains no arbitrary constants. There is only one constant, and that is determined with great accuracy from the ordinary series of hydrogen lines. The second formula contains three arbitrary constants which are selected so as to represent the observed values as nearly as possible. A least square determination was not considered necessary, since the outstanding differences from observation were evidently systematic, and not accidental. The wave-length when m or n is infinite, could be accurately measured, but unfortunately these lines, like those of the ordinary series, do not appear to be present in the stars. The wave-length of the line $H\alpha'$ differs greatly according to the two formulæ, but no means as yet exist for determining radiations of such great wave-length in a star.

EDWARD C. PICKERING.

CAMBRIDGE, U. S.
February 11, 1901.

NOVA PERSEI, NO. 2.[1]

THE early observations made here, of the new star in *Perseus*, are described in *Circular* No. 56. This star may be designated *Nova Persei*, No. 2, to distinguish it from the star in R. A. $1^h\ 55^m.1$, Dec. $+56°\ 15'$, which appeared in this constellation in 1887. A photograph

[1] *Harvard College Observatory Circular* No. 57.

Through an inadvertence in making up the copy from the notebook and an oversight in the correction of the proof, two confusing errors appear in lines 13 to 19, page 140, of my paper "On the Heat Radiation of *Arcturus, Vega, Jupiter, and Saturn*," published in the March number of this JOURNAL. These lines should be corrected to read as follows:

Assuming that the total radiant intensity of unit area of *Arcturus* differs little from that of the Sun,[1] then if $E_a : E_s$ equal the ratio of the heat quantities measured, $\theta_a : \theta_s$ that of the angular diameters, $\theta_a = \theta_s \sqrt{\frac{E_a}{E_s}}$. Further, if D_a and D_s represent the distances of *Arcturus* and the Sun, and V_a and V_s their respective volumes, then $V_a = \frac{V_s D_a^3}{D_s^3} \left(\frac{E_a}{E_s}\right)^{\frac{3}{2}}$. In this equation I believe

The above correction is so printed that it will exactly cover the defective portion of the text as a paster.

E. F. NICHOLS.

of the vicinity of the *Nova*, taken with the Cooke lens on February 19, 1901, with an exposure of 66^m, beginning at $11^h\ 18^m$ G. M. T., is shown in Fig. 1. For comparison, a similar photograph taken on February 26, 1901, with an exposure of 56^m, beginning at $14^h\ 32^m$ G. M. T., and showing the *Nova*, is given in Fig. 2.[1]

The accompanying plate gives enlargements, made with a moving plate, of three photographs of spectra taken with the 11-inch Draper telescope. The first represents the new star in *Perseus* taken on February 22, 1901, with an exposure of 40^m, beginning at $16^h\ 08^m$ G. M. T. The second represents the same star taken on February 24, 1901, with an exposure of 66^m, beginning at $14^h\ 43^m$ G. M. T. The change from the first form of spectrum to the second must have been very sudden. A plate taken through dense clouds on February 23, with an exposure of 29^m, beginning at $11^h\ 37^m$ G. M. T., showed but little change, while Professor Vogel has announced that a photograph taken the same night shows a spectrum traversed by several broad, hazy bands. The third spectrum represents *Nova Aurigæ*, taken on February 5, 1892, with an exposure of 123^m, beginning at $11^h\ 09^m$, G. M. T. It will be seen that the second and third spectra closely resemble each other, but that the lines in *Nova Aurigæ* are much narrower and more sharply defined. The later photographs of the spectrum of the new star in *Perseus* show numerous changes, the dark lines and the edges of the bright lines being, in many cases, well defined.

EDWARD C. PICKERING.

March 15, 1901.

SECOND CHART AND CATALOGUE FOR OBSERVING *NOVA PERSEI*.[2]

THIS second chart and catalogue have been prepared for telescopic observation of the *Nova*. They were both made on the plan of the *Atlas Stellarum Variabilium* (Series III), except that the new star is placed 15′ south of the center of the chart, on account of some bright stars north of the *Nova*, which will be needed for comparison.

An *auxiliary* chart was added to the principal one, to enable those observers who have no circles attached to their telescopes to find the *Nova*. They will experience little difficulty in first setting on δ *Persei*,

[1] These figures are here omitted.— EDS.

[2] The first chart prepared by the Georgetown College Observatory was intended for naked-eye observations of the *Nova*.— EDS.

which is of third magnitude, then passing to ψ and σ, and from there sweeping directly south along the stars indicated on the auxiliary chart, until they reach the stars Nos. 2 and 3 of the principal chart.

On this chart three divisions or squares may be distinguished as regards the *density* of the stars. The large square measures one degree in each coördinate, and contains all the *B. D.* stars (except $+43^{\circ}\!.745$, which is of 10.5 magnitude). In a smaller square, which is $30'$ wide and whose center is the *Nova*, all stars have been entered which are visible in our twelve-inch refractor with a power of about fifty diameters, while in the central square of only $10'$ width fainter stars have been added by means of a magnifying power of about 100 diameters. Great difficulty was experienced in seeing and measuring these objects on account of the brightness of the new star.

The *positions* of all the stars within the square $30'$ wide have been determined by means of a semicircular glass scale, divided into parts $3'$ wide, and of a chronograph. These positions are differential with regard to the *Nova* as zero point. From the preface to the *Atlas*, in which the details of this method are described, we only recall the statement that the Right Ascensions are supposed to be correct within 1^s, while the Declinations may be erroneous by $0'\!.3$ or even $0'\!.6$. The positions of the stars outside this square have been computed from the *B. D.* or the *A.G.C.*, assuming the position of the *Nova*, given in the title of the chart, to be correct for the beginning of the year 1901.

The *magnitudes* of all the stars on the chart were determined by sequences of steps based on at least two independent estimates. The formula for transforming these steps to the *B. D.* scale is of the same character as those in the *Atlas*. It will be an easy matter to replace it by any other process of transformation, graphical or arithmetical, when a scale of standard magnitudes is determined by photometric means. The faintest stars near the *Nova* could not be properly estimated on account of the brightness of the latter.

It is well to remark that only three good nights (March 7, 14, 16) were available for making this chart, and that then the *Nova* was brighter than fourth magnitude. Yet it has seemed better to distribute chart and catalogue as they are without delay, in order to facilitate observation. A more accurate scale of magnitudes, for the final reduction of the observations, may be determined at any future time, when the brightness of the new star shall have faded away.

CHART II.

NOVA PERSEI.

$3^h\ 24^m\ 28^s$; $+43°\ 33'.9$.

No.	Steps	Mag.	*B. D.*		$\Delta\alpha$	$\Delta\delta$	Notes
1	0	$7^m.1$	$7^m.0$	+43° 730	$-2^m\ 55^s$	+27'.9	*H. P.* $6^m.9$; red.
2	10	7.4	6.5	44 734	+1 22	+57.3	6.5
3	17	7.5	7.5	44 732	+1 0	+56.3	7.3
4	21	7.6	7.3	44 742	+3 25	+54.4	7.8
5	23	7.7	7.5	43 732	−2 37	− 9.4	7.3
6	25	7.8	7.6	43 720	−4 15	−15.5	
7	31	7.9	8.0	43 766	+4 7	− 2.8	
8	37	8.1	8.4	43 728	−3 10	+28.7	
9	42	8.2	8.5	44 717	−0 41	+55.6	
10	46	8.3	8.9	43 726	−3 29	+15.8	
11	49	8.4	8.6	43 744	+0 50	−13.8	
12	52	8.5	8.7	43 729	−3 10	+25.7	
13	55	8.6	8.9	44 712	−2 32	+44.3	
14	59	8.7	9.1	44 741	+3 20	+41.3	
15	59	8.7	9.1	43 723	−3 51	−11.4	
16	61	8.7	8.9	44 721	+0 4	+45.0	
17	63	8.8	8.8	43 740	−0 7	+18.3	
18	65	8.8	9.1	43 746	+1 10	−22.8	
19	65	8.8	9.0	44 724	+0 23	+37.8	
20	67	8.9	9.0	43 739	−0 21	+ 4.8	
21	70	9.0	9.0	43 749	+1 41	− 8.4	
22	70	9.0	9.0	43 751	+1 50	−17.9	
23	72	9.0	9.1	43 733	−2 29	− 3.9	
24	73	9.0	9.0	43 748	+1 34	+20.1	
25	73	9.0	9.1	43 731	−2 49	+ 4.9	
26	75	9.1	9.0	43 758	+3 1	− 9.3	
27	75	9.1	9.0	43 742	+0 37	+29.9	
28	79	9.2	9.0	43 757	+2 39	− 2.9	
29	79	9.2	9.1	43 741	+0 28	+19.4	
30	80	9.2	9.2	43 759	+3 4	+22.5	
31	82	9.3	9.1	43 752	+1 58	+15.0	
32	84	9.4	9.2	43 743	+0 43	+ 8.7	
33	89	9.5	9.4	43 760	+3 20	+ 3.2	
34	92	9.6	9.5	43 735	−1 5	+ 3.4	
35	93	9.6	9.5	43 755	+2 27	+ 4.4	
36	94	9.6	9.5	43 734	−2 9	+16.6	
37	96	9.7	9.5	43 738	−0 28	−12.3	
38	102	9.8	9.5	43 737	−0 46	− 9.6	
39	105	9.9	9.5	43 756	+2 36	− 2.7	
40	107	10.0	9.5	43 753	+2 21	+20.1	
41	108	10.0	9.5	43 750	+1 47	−10.2	
42	111	10.1			+0 35	+ 0.2	
43	117	10.2			−1 32	+ 3.0	
44	120	10.3			0 0	−10.9	
45	124	10.4			−1 24	+ 9.3	

Mag. = 9.0 + 0.027 (St. − 71.1).

CHART II. — *Continued.*

NOVA PERSEI.

$3^h\ 24^m\ 28^s$; $+43°\ 33'.9$.

No.	Steps	Mag.	BD.		Δα	Δδ	Notes
46	133	$10^m.7$			−1 22	+ 6.3	
47	134	10.7			+0 54	+11.9	
48	138	10.8			+1 1	− 5.0	
49	145	11.0			−0 24	− 1.5	
50	148	11.1			+0 31	+11.4	
51	153	11.2			−1 17	+ 3.0	
52	153	11.2			−1 21	+ 9.4	
53	158	11.3			−0 32	+14.4	
54	159	11.4			−0 21	− 3.9	
55	160	11.4			+0 36	−14.4	
56	161	11.4			−0 17	+ 2.4	
57	167	11.6			−0 6	+11.5	
58	168	11.6			−0 45	− 0.8	
59	170	11.6			+1 13	−12.0	
60	171	11.7			−0 52	+14.1	
61	172	11.7			−1 1	− 9.0	
62	175	11.8			+0 3	+12.6	
63	177	11.9			−0 49	+12.3	
64	177	11.9			+0 45	− 3.7	
65	178	11.9			+0 42	+ 4.8	
66	179	11.9			+0 33	− 4.1	
67	182	12.0			−0 58	− 8.3	
68	185	12.1			+0 34	+ 1.5	
69		(12)			+0 3	− 0.5	elongated.[1]
70	186	12.1			+1 11	−10.0	
71	193	12.3			+0 26	− 8.1	
72	196	12.4			+1 9	− 9.7	
73	196	12.4			−0 7	− 6.6	
74	199	12.4			+0 21	+ 1.6	
75	200	12.5			+0 17	−12.0	
76	205	12.6			+0 20	−11.2	
77	209	12.7			+0 13	+ 1.0	
78	212	12.8			−0 14	+ 1.6	
79	212	12.8			+0 13	− 0.7	
80		(13)			−0 3	− 0.5	
81		(13-14)			+0 9	− 0.4	

Mag. = 9.0 + 0.027 (St. − 71.1).

JOHN G. HAGEN, S. J.

WASHINGTON, D. C.,
March 19, 1901.

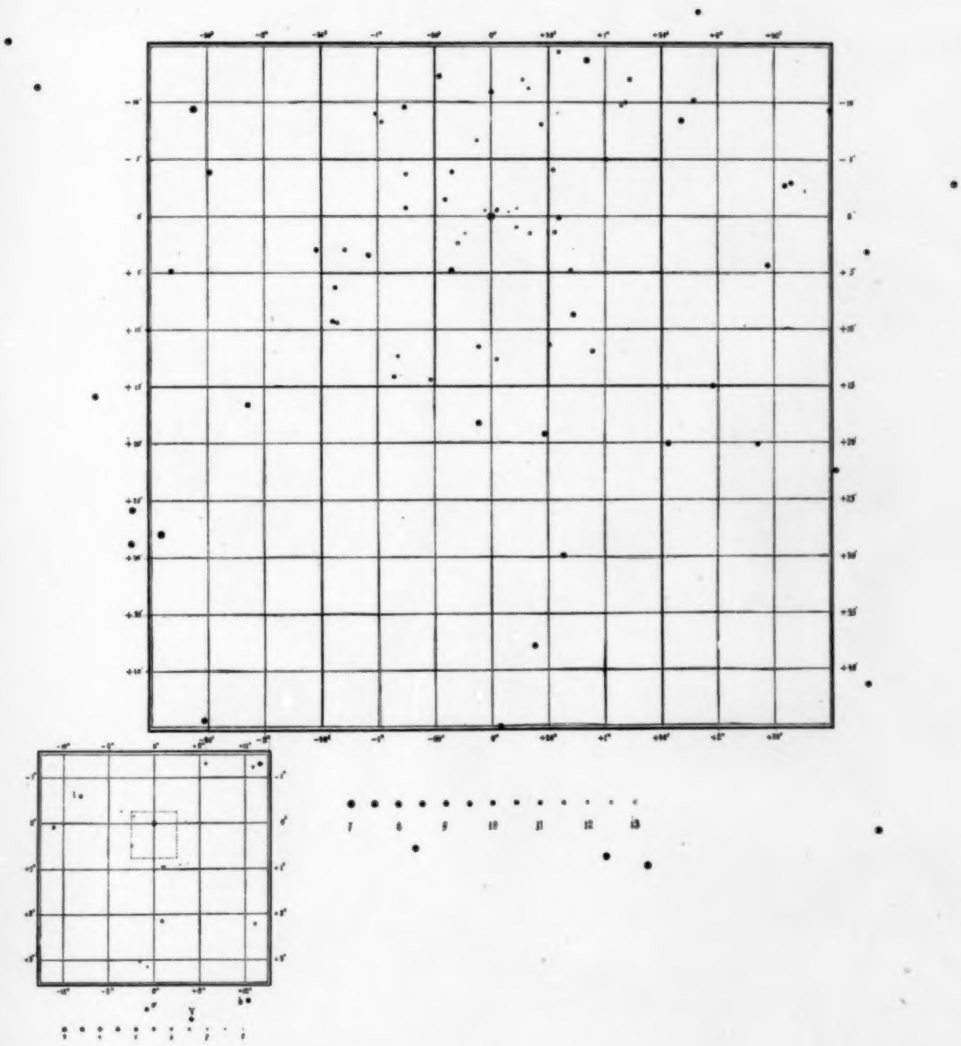
[1] Professor T. J. J. See kindly examined this object with the 26-inch refractor and found it to consist of two stars which he estimated as 14th magnitude each.

GEORGETOWN COLLEGE OBSERVATORY

Chart II

Nova Persei

$3^h\ 24^m\ 28^s : +43^\circ\ 33'.9$



CHANGES IN THE SPECTRUM OF *NOVA PERSEI*.[1]

Since the publication of *Bulletin* No. 16 the weather has been unusually cloudy and comparatively few observations of *Nova Persei* have been secured. The following observations have been obtained by Mr. Parkhurst. With the exception of those marked "vis." they were all made with the wedge photometer referred to in the last *Bulletin*.

		Gr. M. T.	Mag.				Gr. M. T.	Mag.	
1901	Feb.	25.52	1.0	vis.	1901	Mar.	14.56	3.5	vis.
		26.52	1.1	vis.			15.56	3.72	
		27.56	2.0	vis.			16.6	3.69	
		28.58	1.90				17.56	3.50	
	Mar.	3.56	2.71				22.59	4.08	
		4.58	2.8	vis.			31.58	4.20	
		5.54	2.71			Apr.	3.60	5.39	
		6.56	3.08				8.58	4.14	
		7.58	3.2	vis.			9.56	4.26	
		11.56	3.62				10.58	5.48	
		12.56	3.30						

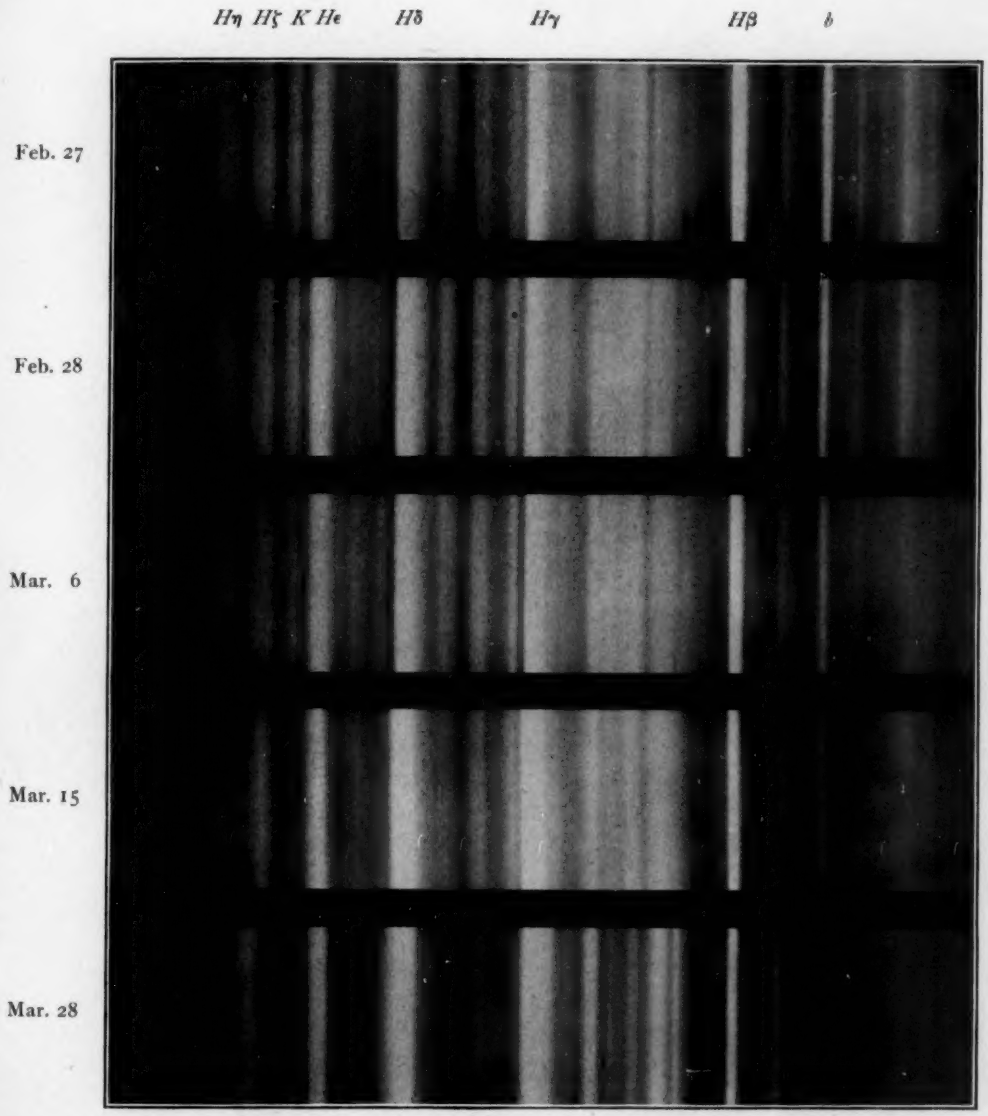
The magnitudes are based on the system of the Harvard *Photometric Durchmusterung* (*H. C. O. Annals*, XLV). There seems to be no doubt of the reality of the considerable fluctuations shown, and from the internal agreement of the measures it is probable that the amount of these fluctuations is represented within one or two tenths of a magnitude.

Since the publication of the previous list Mr. Ellerman has obtained photographs of the spectrum as follows:

Date			No. of Plates	Dispersion	Region
1901	March	15	3	3 prisms	D to 4400
		15	1	3 prisms	5000 to 4200
		15	2	1 prism	5700 to 3700
		22	2	3 prisms	D to 4400
		22	1	1 prism	5700 to 3700
		28	1	3 prisms	D to 4400
		28	1	1 prism	5700 to 3700
		31	1	3 prism	D to 4400
	April	3	2	1 prisms	5700 to 3700
		8	1	3 prisms	D to 4400
		8	2	1 prism	5700 to 3700
		9	1	1 "	5700 to 3700
		10	1	1 "	*Hα* to 4000

[1] *Yerkes Observatory Bulletin* No. 17.

PLATE V



SPECTRUM OF *NOVA PERSEI*

PHOTOGRAPHED WITH THE 40-INCH YERKES TELESCOPE BY FERDINAND ELLERMAN

The changes which have taken place are well illustrated in the plate, which is a reproduction of direct photographic enlargements from the original negatives. On account of the cloudy weather the intervals between some of the photographs are so great that the progressive change is not always fully indicated. In general it will be noticed that the brightness of the spectrum has decreased in both the yellow and the ultra-violet. The decreased intensity of the less refrangible region is particularly striking. It should be remarked that most of the photographs reproduced in the plate were shaded in the process of enlargement so as to bring out the details as well as possible in both the brighter and the fainter regions. For this and other obvious reasons the plate must not be taken to represent the relative intensities of the various portions of the spectrum. If the yellow region in the later photographs had not been shaded in copying it would have been entirely lost in the reproduction on account of its great relative faintness.

In addition to this falling off in intensity in the less refrangible region, the continuous spectrum as a whole has become much fainter relatively to the bright lines. Exception must apparently be made of the spectrum as photographed April 8, unfortunately too late for reproduction on the plate. On this evening, as the photometric measures show, the *Nova* was brighter than on April 3. This increase in magnitude was accompanied by marked brightening of the continuous spectrum relatively to the bright lines. The $H\zeta$ line, which on the photograph of March 28 seems to have shifted toward the violet, is shown on the photograph of April 8 in its original position, but greatly decreased in intensity.

There have been many changes of importance in the relative intensities of the bright lines. The *b* line, which was so conspicuous in the earlier photographs, has greatly decreased in intensity; the K line of calcium has undergone a similar change in brightness and now seems to have disappeared entirely; $H\beta$ has become narrower and sharper and the relative intensities of its several components have undergone marked variations. The plate will serve to show in a general way the changes which the other lines have experienced. The spectrum photographed on March 28 is in some respects the most remarkable of the series on account of the apparent shifting of several of the hydrogen lines and the rise into prominence of lines which were previously inconspicuous.

The present *Bulletin* is intended merely to call attention to the

more striking changes which have taken place in the spectrum of the *Nova*. Illustrations of other changes will be published later. It should be added, however, that these changes include a duplication of the dark lines on the more refrangible edge of the bright hydrogen lines; these were at first rather broad and poorly defined, but subsequently, March 15, became sharp double lines; they have recently become much fainter and are no longer double.

Special attention has been given to the two dark D lines and the bright band upon which they were projected. This bright band has gradually moved toward the violet so that the two narrow dark lines which were at first nearly central on the band (see Fig. 1, Plate III, *Bulletin* No. 16) are now at its less refrangible edge. The dark line on the more refrangible edge of this band, which in *Bulletin* No. 16, was provisionally designated D_3 has given place to a much broader, but fainter, band extending toward the violet.

Various laboratory investigations on the spectrum of the spark and arc in air and in certain liquids will be described in a later paper. With the arc taken in air between carbon poles moistened from time to time with a solution of sodium hyposulphite, the appearance of the sodium lines is almost precisely like that presented by the *Nova* on February 28. The narrow dark lines due to the absorption of the cooler sodium vapor in the outer part of the arc are superposed upon a very broad bright band like that in the spectrum of the *Nova*. The experiments with the arc and spark will be continued with more powerful apparatus.

April 11, 1901. GEORGE E. HALE.

ULTRA-VIOLET CORONAL LINES.

IN the number of this JOURNAL for November 1900 which came to hand only last week I find M. Deslandres announcing the discovery of "two complete rings due to two new coronal radiations" in the ultra-violet as a result of his observations of the total solar eclipse of May last (page 288).

I hasten therefore to state that these two ultra-violet rings were obtained by me during the Indian eclipse of 1898 with a prismatic camera composed of two spar prisms of 60° angle and about 1 inch face and a single quartz lens of about 24 inches focus. As a *first approximation* only, the wave-lengths of the radiations came out 3456 and 3391, but I hope very soon to obtain a more correct determination.

K. D. NAEGAMVALA.

POONA, March 1, 1901.

PLATE VI

HENRY AUGUSTUS ROWLAND